

LEARNING MADE EASY

3rd Edition



Physics I

WORKBOOK

WITH
ONLINE
PRACTICE



for
dummies[®]
A Wiley Brand

Practice on hundreds
of physics problems

Access online chapter quizzes
to review key physics concepts

Get complete answers
for all problems





Physics I Workbook

3rd Edition with Online Practice

for
dummies[®]
A Wiley Brand

Physics I Workbook For Dummies[®], 3rd Edition with Online Practice

Published by: **John Wiley & Sons, Inc.**, 111 River Street, Hoboken, NJ 07030-5774, www.wiley.com

Copyright © 2022 by John Wiley & Sons, Inc., Hoboken, New Jersey

Published simultaneously in Canada

No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise, except as permitted under Sections 107 or 108 of the 1976 United States Copyright Act, without the prior written permission of the Publisher. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008, or online at <http://www.wiley.com/go/permissions>.

Trademarks: Wiley, For Dummies, the Dummies Man logo, Dummies.com, Making Everything Easier, and related trade dress are trademarks or registered trademarks of John Wiley & Sons, Inc. and may not be used without written permission. All other trademarks are the property of their respective owners. John Wiley & Sons, Inc. is not associated with any product or vendor mentioned in this book.

LIMIT OF LIABILITY/DISCLAIMER OF WARRANTY: WHILE THE PUBLISHER AND AUTHORS HAVE USED THEIR BEST EFFORTS IN PREPARING THIS WORK, THEY MAKE NO REPRESENTATIONS OR WARRANTIES WITH RESPECT TO THE ACCURACY OR COMPLETENESS OF THE CONTENTS OF THIS WORK AND SPECIFICALLY DISCLAIM ALL WARRANTIES, INCLUDING WITHOUT LIMITATION ANY IMPLIED WARRANTIES OF MERCHANTABILITY OR FITNESS FOR A PARTICULAR PURPOSE. NO WARRANTY MAY BE CREATED OR EXTENDED BY SALES REPRESENTATIVES, WRITTEN SALES MATERIALS OR PROMOTIONAL STATEMENTS FOR THIS WORK. THE FACT THAT AN ORGANIZATION, WEBSITE, OR PRODUCT IS REFERRED TO IN THIS WORK AS A CITATION AND/OR POTENTIAL SOURCE OF FURTHER INFORMATION DOES NOT MEAN THAT THE PUBLISHER AND AUTHORS ENDORSE THE INFORMATION OR SERVICES THE ORGANIZATION, WEBSITE, OR PRODUCT MAY PROVIDE OR RECOMMENDATIONS IT MAY MAKE. THIS WORK IS SOLD WITH THE UNDERSTANDING THAT THE PUBLISHER IS NOT ENGAGED IN RENDERING PROFESSIONAL SERVICES. THE ADVICE AND STRATEGIES CONTAINED HEREIN MAY NOT BE SUITABLE FOR YOUR SITUATION. YOU SHOULD CONSULT WITH A SPECIALIST WHERE APPROPRIATE. FURTHER, READERS SHOULD BE AWARE THAT WEBSITES LISTED IN THIS WORK MAY HAVE CHANGED OR DISAPPEARED BETWEEN WHEN THIS WORK WAS WRITTEN AND WHEN IT IS READ. NEITHER THE PUBLISHER NOR AUTHORS SHALL BE LIABLE FOR ANY LOSS OF PROFIT OR ANY OTHER COMMERCIAL DAMAGES, INCLUDING BUT NOT LIMITED TO SPECIAL, INCIDENTAL, CONSEQUENTIAL, OR OTHER DAMAGES.

For general information on our other products and services, please contact our Customer Care Department within the U.S. at 877-762-2974, outside the U.S. at 317-572-3993, or fax 317-572-4002. For technical support, please visit <https://hub.wiley.com/community/support/dummies>.

Wiley publishes in a variety of print and electronic formats and by print-on-demand. Some material included with standard print versions of this book may not be included in e-books or in print-on-demand. If this book refers to media such as a CD or DVD that is not included in the version you purchased, you may download this material at <http://booksupport.wiley.com>. For more information about Wiley products, visit www.wiley.com.

Library of Congress Control Number: 2021949599

ISBN 978-1-119-71647-1; ISBN 978-1-119-71648-8 (ebk); ISBN 978-1-119-71652-5 (ebk)

Contents at a Glance

Introduction	1
Part 1: Getting Started with Physics	5
CHAPTER 1: Reviewing Physics Basics	7
CHAPTER 2: The Big Three: Displacement, Velocity, and Acceleration	25
CHAPTER 3: Vectors: Knowing Where You're Headed	41
Part 2: May the Forces Be with You	59
CHAPTER 4: Applying Force	61
CHAPTER 5: Working with Inclined Planes	81
CHAPTER 6: Round and Round: Circular Motion	101
Part 3: Being Energetic: Work	121
CHAPTER 7: Working the Physics Way	123
CHAPTER 8: Getting Things to Move: Momentum and Kinetic Energy	147
CHAPTER 9: Winding It Up: Rotational Motion and Torque	167
CHAPTER 10: Getting Dizzy with Rotational Dynamics	185
CHAPTER 11: Simple Harmonic Motion	203
Part 4: Obeying the Laws of Thermodynamics	223
CHAPTER 12: You're Getting Warm: Thermodynamics	225
CHAPTER 13: Under Pressure: From Solid to Liquid to Gas	243
CHAPTER 14: All about Heat and Work	261
Part 5: Zap: Electricity	283
CHAPTER 15: Static Electricity: Electrons at Rest	285
CHAPTER 16: Electrons in Motion: Circuits	305
Part 6: The Part of Tens	327
CHAPTER 17: Ten Common Mistakes People Make When Solving Problems	329
CHAPTER 18: Ten Wild Physics Theories	333
Index	337

Table of Contents

INTRODUCTION	1
About This Book	1
Foolish Assumptions	1
Icons Used in This Book	2
Beyond the Book	2
Where to Go from Here	3
PART 1: GETTING STARTED WITH PHYSICS	5
CHAPTER 1: Reviewing Physics Basics	7
Measuring the Universe	7
Putting Scientific Notation to Work	10
Converting between Units	11
Converting through Multiple Units	14
Converting Times	15
Counting Significant Figures	16
Coming Prepared with Some Algebra	18
Being Prepared with Trigonometry	19
Answers to Problems about Physics Basics	21
CHAPTER 2: The Big Three: Displacement, Velocity, and Acceleration	25
From Point A to B: Displacement	25
Reading That Speedometer	27
Putting Pedal to Metal: Acceleration	29
Connecting Acceleration, Time, and Displacement	31
Connecting Velocity, Acceleration, and Displacement	33
Answers to Problems about Displacement, Velocity, and Acceleration	35
CHAPTER 3: Vectors: Knowing Where You're Headed	41
Creating a Vector	41
Understanding Vector Components	43
Finding a Vector's Components	45
Finding a Vector's Magnitude and Direction	46
Adding Vectors Together	48
Handling Velocity as a Vector	51
Answers to Problems about Vectors	54
PART 2: MAY THE FORCES BE WITH YOU	59
CHAPTER 4: Applying Force	61
Newton's First Law of Motion	61
Newton's Second Law of Motion	62
Force Is a Vector	65

	Calculating Net Force and Acceleration	67
	Sorting Out Weight and Mass	69
	The Balancing Act of Equilibrium.	71
	Newton's Third Law of Motion.	73
	Answers to Problems about Force	74
CHAPTER 5:	Working with Inclined Planes.	81
	Breaking Ramps Up into Vectors.	81
	Acceleration and Inclined Planes.	84
	Running Down Ramps: Speed	85
	Friction on Inclined Planes	87
	Starting from zero: Static friction.	88
	Already in motion: Kinetic friction.	89
	Static Friction along Ramps	90
	Kinetic Friction along Ramps	91
	Acceleration along Ramps Including Friction.	93
	Answers to Problems about Inclined Planes	95
CHAPTER 6:	Round and Round: Circular Motion.	101
	Converting between Angles	101
	Period and Frequency	103
	Getting into Angular Velocity	104
	Whipping Around with Angular Acceleration.	106
	Connecting Angular Velocity and Angular Acceleration to Angles	108
	Connecting Angular Acceleration and Angle to Angular Velocity	109
	Handling Centripetal Acceleration	110
	Getting Forceful: Centripetal Force.	112
	Answers to Problems about Circular Motion.	114
	PART 3: BEING ENERGETIC: WORK.	121
CHAPTER 7:	Working the Physics Way.	123
	A Different Kind of Work.	124
	Dealing with the Net Force	125
	Getting Energetic: Kinetic Energy	127
	Getting Kinetic Energy from Work.	129
	Storing Your Energy: Potential Energy	132
	Powering It Up	134
	Answers to Problems about Work.	136
CHAPTER 8:	Getting Things to Move: Momentum and Kinetic Energy.	147
	Acting on Impulse	147
	Getting Some Momentum	149
	Relating Impulse and Momentum.	150
	Conserving Momentum	152
	Conserving Kinetic Energy — or Not.	154
	Collisions in Two Dimensions	156
	Answers to Problems about Momentum and Kinetic Energy	159

CHAPTER 9: Winding It Up: Rotational Motion and Torque	167
Finding Tangential Speed	167
Targeting Tangential Acceleration	170
Angular Velocity as a Vector	171
Angular Acceleration as a Vector	172
Doing the Twist: Torque	173
The Balancing Act: Rotational Equilibrium	176
Answers to Problems about Rotational Motion and Torque	180
CHAPTER 10: Getting Dizzy with Rotational Dynamics	185
Putting Newton on Wheels	185
Moments of Inertia for Everyone	187
Doing Some Rotational Work	190
Round and Round: Rotational Kinetic Energy	191
Working with Ramps Again	193
Can't Stop This: Angular Momentum	195
Answers to Problems about Rotational Dynamics	197
CHAPTER 11: Simple Harmonic Motion	203
Hooking into Hooke's Law	203
Simply Simple Harmonic Motion	205
Getting Periodic	207
Considering Velocity	208
Figuring the Acceleration	210
Bouncing Around with Springs	211
Talking about Energy	213
Following the Ticktock of Pendulums	214
Answers to Problems about Simple Harmonic Motion	216
PART 4: OBEYING THE LAWS OF THERMODYNAMICS	223
CHAPTER 12: You're Getting Warm: Thermodynamics	225
Converting between Temperature Scales	225
Getting Bigger: Linear Expansion	227
Plumping It Up: Volume Expansion	230
Getting Specific with Heat Capacity	231
Changes of Phase: Latent Heat	234
Answers to Problems about Thermodynamics	236
CHAPTER 13: Under Pressure: From Solid to Liquid to Gas	243
How Heat Flows: Convection	244
How Heat Is Produced: Conduction	245
How Heat Is Produced: Radiation	248
A Biggie: Avogadro's Number	250
Ideally Speaking: The Ideal Gas Law	251
Molecules in Motion	253
Answers to Problems about Pressure	255

CHAPTER 14: All about Heat and Work	261
The First Law of Thermodynamics	261
Constant Pressure: Isobaric Processes	262
Constant Volume: Isochoric Processes	265
Constant Temperature: Isothermal Processes	266
At Constant Heat: Adiabatic	268
The Direction of Heat: The Second Law of Thermodynamics	271
Making Heat Work: Heat Engines	271
Maximum Efficiency: Carnot Heat Engines	274
The Third Law of Thermodynamics	275
Answers to Problems about Heat and Work	276
PART 5: ZAP: ELECTRICITY	283
CHAPTER 15: Static Electricity: Electrons at Rest	285
Talking about Electric Charges	285
Getting Forceful with Charges	286
Electrical Forces Are Vectors	288
Force at a Distance: Electric Fields	289
Easy Electric Field: Parallel Plate Capacitors	291
Ramping Up Some Voltage	293
Electric Potential from Point Charges	294
Answers to Problems about Static Electricity	297
CHAPTER 16: Electrons in Motion: Circuits	305
Electrons in a Whirl: Current	305
Giving You Some Resistance: Ohm's Law	306
Powering It Up	308
One after the Other: Series Circuits	309
All for One: Parallel Circuits	311
The Whole Story: Kirchhoff's Rules	314
Answers to Problems about Circuits	317
PART 6: THE PART OF TENS	327
CHAPTER 17: Ten Common Mistakes People Make When Solving Problems	329
Mixing Units	329
Expressing the Answer in the Wrong Units	329
Swapping Radians and Degrees	330
Getting Sines and Cosines Mixed Up	330
Failing to Treat Vectors as Vectors	330
Neglecting Latent Heat	330

Getting the Direction of Forces Wrong	331
Getting the Signs Wrong in Kirchhoff Loops	331
Adding Resistors Incorrectly.	332
Using the Wrong Temperature in the Ideal Gas Law	332
CHAPTER 18: Ten Wild Physics Theories	333
The Universal Speed Limit	333
Through the Looking Glass, and What Chien-Shiung Found There	334
Wanted: Dead and Alive	334
Quantum Objects Can Tunnel	334
Mass Is a Kind of Energy.	334
Vacuum Is Not Just Empty Space.	335
“Constants” in Physics Change.	335
Stuck in the Middle of a Proton	335
The Expansion of the Universe Is Accelerating	336
Some Things Never Change	336
INDEX	337

Introduction

Physics is about the world and everything in it. Physics describes that world and the kinds of things that take place in it. Sometimes, however, physics seems like an imposition from outside — a requirement you have to get through.

That's a shame, because it's your world that physics describes. Under the burden of physics problems, though, things can get tough. That's where this book comes in, because it's designed to let you tackle those problems with ease.

Kirchhoff's laws? No problem. Carnot engines? No worries. This book addresses these topics and more. After you're done reading, you'll be a problem-solving pro.

About This Book

This book is crammed with physics example problems and practice questions that are designed to show you solutions for the kinds of problems you may encounter in physics classes. And when you see how the solutions to problems are done step by step, solving similar problems should be a breeze.

Many books have endless conventions that you have to learn before you can start reading. Not this one. In fact, all you need to know is that new terms are given in italics, like *this*, when they're introduced. You should also know that vectors, which are those items that have both a magnitude and a direction, are given in bold, like this: **B**.

Foolish Assumptions

We're assuming that you're using this book in conjunction with a physics class or textbook, because this book keeps the derivation of physical formulas to a minimum. The emphasis here is on solving problems, not deriving formulas. So some knowledge of the physics you're going to be using here is helpful. This book is designed to help you with the nitty-gritty, not to introduce the topics from scratch.

You should also know some algebra. You don't need to be an algebra pro, but you should know how to move items from one side of an equation to another and how to solve for values. Take a look at the discussion in Chapter 1 if you're unsure.

You also need a little knowledge of trigonometry, but not much. Again, take a look at the discussion in Chapter 1, where all the trig you need to know — a grasp of sine and cosine — is reviewed in full.

Icons Used in This Book

You find a few icons in this book, and here's what they mean:



TIP

This icon points out helpful hints, ideas, or shortcuts that save you time or that give you alternative ways to think about a particular concept.



REMEMBER

This icon marks something to remember, such as a law of physics or a particularly juicy equation.



TECHNICAL
STUFF

This icon means that what follows is technical, insider stuff. You don't have to read it if you don't want to, but if you want to become a physics pro (and who doesn't?), take a look.



EXAMPLE

This icon highlights examples that show you how to work each type of problem.

Beyond the Book

In addition to what you're reading right now, this book has a free access-anywhere Cheat Sheet for when you need a quick physics refresher. To get this Cheat Sheet, simply go to www.dummies.com and type **Physics I Workbook For Dummies Cheat Sheet** in the Search box. There you'll find common mistakes to avoid, the values of important constants, and equations to remember.

This book also comes with over 200 online practice questions that you can use to test your knowledge of different physics topics. To gain access to the online practice, all you have to do is register. Just follow these simple steps:

1. **Register your book or ebook at Dummies.com to get your PIN. Go to www.dummies.com/go/getaccess.**
2. **Select your product from the dropdown list on that page.**
3. **Follow the prompts to validate your product, and then check your email for a confirmation message that includes your PIN and instructions for logging in.**

If you do not receive this email within two hours, please check your spam folder before contacting us through our Technical Support website at <http://support.wiley.com> or by phone at 877-762-2974.

Now you're ready to go! You can come back to the practice material as often as you want — simply log on with the username and password you created during your initial login. No need to enter the access code a second time.

Your registration is good for one year from the day you activate your PIN.

Where to Go from Here

You're ready to jump into Chapter 1. You don't have to start there, of course. You can read this book in any order you like instead of reading it from beginning to end. But if you want some important, general, problem-solving background, take a look at Chapter 1 first.

1

Getting Started with Physics

IN THIS PART . . .

Grasp fundamental physics measurements, scientific notation, and converting among units, distances, and time.

Master the motion of displacement, velocity, and acceleration.

Point yourself in the right direction with vectors.

IN THIS CHAPTER

- » Laying down measurements
- » Simplifying with scientific notation
- » Practicing conversions
- » Drawing on algebra and trigonometry

Chapter **1**

Reviewing Physics Basics

This chapter gets the ball rolling by discussing some fundamental physics measurements. At its root, physics is all about making measurements (and using those measurements as the basis of predictions), so it's the perfect place to start! As you go through the process of converting measurements from one unit to another, you'll practice applying math skills to physics problems.

Measuring the Universe

A great deal of physics has to do with making measurements — that's the way all physics gets started. For that reason, physics uses a number of measurement systems, such as the CGS (centimeter-gram-second) system and the MKS (meter-kilogram-second) system. You also use the standard English system of inches and feet and so on — that's the FPI (foot-pound-inch) system.



REMEMBER

In physics, most measurements have units, such as meters or seconds. For example, when you measure how far and how fast a hockey puck slid, you need to measure both the distance in centimeters and the time in seconds.

For reference, Table 1-1 gives you the primary units of measurement in the MKS system.

Table 1-1 **MKS Units of Measurement**

Measurement	Unit	Abbreviation
Length	meter	m
Mass	kilogram	kg
Time	second	s or sec
Force	newton	N
Energy	joule	J
Pressure	pascal	P
Electric current	ampere	A
Magnetism	tesla	T
Electric charge	coulomb	C

These are the measuring sticks that will become familiar to you as you solve problems and triumph over the math in this workbook. Also for reference, Table 1-2 shows the primary units of measurement (and their abbreviations) in the CGS system. (Don't bother memorizing the ones you're not familiar with now; you can come back to them later as needed.)

Table 1-2 **CGS Units of Measurement**

Measurement	Unit	Abbreviation
Length	centimeter	cm
Mass	gram	g
Time	second	s or sec
Force	dyne	dyn
Energy	erg	erg
Pressure	barye	Ba
Electric current	biot	Bi
Magnetism	gauss	G
Electric charge	franklin	Fr



EXAMPLE

Q. You're told to measure the length of a racecar track using the MKS system. What unit(s) will your measurement be in?

A. The correct answer is meters. The unit of length in the MKS system is the meter.

1 You're told to measure the mass of a marble using the CGS system. What unit(s) will your measurement be in?

2 You're asked to measure the time it takes the moon to circle the Earth using the MKS system. What will your measurement's units be?

3 You need to measure the force a tire exerts on the road as it's moving using the MKS system. What are the units of your answer?

4 You're asked to measure the amount of energy released by a firecracker when it explodes using the CGS system. What are the units of your answer?

7 What is 0.0000056 in scientific notation?

8 What is 6,700.0 in scientific notation?

Converting between Units

Physics problems frequently ask you to convert between different units of measurement. For example, you may measure the number of feet your toy car goes in three minutes and thus be able to calculate the speed of the car in feet per minute, but that's not a standard unit of measure, so you need to convert feet per minute to miles per hour, or meters per second, or whatever the physics problem asks for.

For another example, suppose you have 180 seconds — how much is that in minutes? You know that there are 60 seconds in a minute, so 180 seconds equals three minutes. Here are some common conversions between units:

»» 1 m = 100 cm = 1,000 mm (millimeters)

»» 1 km (kilometer) = 1,000 m

»» 1 kg (kilogram) = 1,000 g (grams)

»» 1 N (newton) = 10^5 dynes

»» 1 J (joule) = 10^7 ergs

»» 1 Pa (pascal) = 10 Ba

»» 1 A (amp) = 0.1 Bi

»» 1 T (tesla) = 10^4 G (gauss)

»» 1 C (coulomb) = 2.9979×10^9 Fr

The conversion between CGS and MKS almost always involves factors of 10 only, so converting between the two is simple. But what about converting to and from the FPI and other systems of measurement? Here are some handy conversions that you can come back to as needed:

»» **Length:**

- 1 m = 100 cm
- 1 km = 1,000 m
- 1 in (inch) = 2.54 cm
- 1 m = 39.37 in
- 1 mile = 5,280 ft = 1.609 km
- 1 Å (angstrom) = 10^{-10} m

»» **Mass:**

- 1 kg = 1,000 g
- 1 slug = 14.59 kg
- 1 u (atomic mass unit) = 1.6605×10^{-27} kg

»» **Force:**

- 1 lb (pound) = 4.448 N
- 1 N = 10^5 dynes
- 1 N = 0.2248 lb

»» **Energy:**

- 1 J = 10^7 ergs
- 1 J = 0.7376 ft-lb
- 1 BTU (British thermal unit) = 1,055 J
- 1 kWh (kilowatt hour) = 3.600×10^6 J
- 1 eV (electron volt) = 1.602×10^{-19} J

»» **Power:**

- 1 hp (horsepower) = 550 ft-lb/s
- 1 W (watt) = 0.7376 ft-lb/s

Because conversions are such an important part of physics problems, and because you have to keep track of them so carefully, there's a systematic way of handling conversions: You multiply by a conversion constant that equals 1, such that the units you don't want cancel out.



EXAMPLE

Q. A ball drops 5 meters. How many centimeters did it drop?

A. The correct answer is 500 centimeters. To perform the conversion, you do the following calculation:

$$5.0 \text{ meters} \times \frac{100 \text{ centimeters}}{1 \text{ meter}} = 500 \text{ centimeters}$$

Note that 100 centimeters divided by 1 meter equals 1 because there are 100 centimeters in a meter. In the calculation, the units you don't want — meters — cancel out.

9 How many centimeters are in 2.35 meters?

10 How many seconds are in 1.25 minutes?

11 How many inches are in 2.0 meters?

12 How many grams are in 3.25 kg?

Converting through Multiple Units

Sometimes you have to make multiple conversions to get what you want. That demands multiple conversion factors. For example, if you want to convert from inches to meters, you can use the conversion that 2.54 centimeters equals 1 inch — but then you have to convert from centimeters to meters, which means using another conversion factor.

Try your hand at this example question that involves multiple conversions:



Q. Convert 10 inches into meters.

A. The correct answer is 0.254 m.

EXAMPLE

1. You know that 1 inch = 2.54 centimeters, so start with that conversion factor and convert 10 inches into centimeters:

$$10 \text{ in} \times \frac{2.54 \text{ cm}}{1 \text{ in}} = 25.4 \text{ cm}$$

2. Convert 25.4 cm into meters by using a second conversion factor:

$$25.4 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}} = 0.254 \text{ m}$$

13 Given that there are 2.54 centimeters in 1 inch, how many centimeters are there in 1 yard?

14 How many centimeters are in a kilometer?

15 How many inches are in an angstrom, given that 1 angstrom (\AA) = 10^{-8} cm?

16 How many inches are in 3.0 meters, given that there are 2.54 cm in 1 inch?

Converting Times

Physics problems frequently ask you to convert between different units of time: seconds, minutes, hours, and even years. These times involve all kinds of calculations because measurements in physics books are usually in seconds, but can frequently be in hours.



EXAMPLE

Q. An SUV is traveling 2.78×10^{-2} kilometers per second. What's that in kilometers per hour?

A. The correct answer is 100 km/hr.

1. You know that there are 60 minutes in an hour, so start by converting from kilometers per second to kilometers per minute:

$$2.78 \times 10^{-2} \frac{\text{km}}{\text{sec}} \times \frac{60 \text{ sec}}{1 \text{ minute}} = 1.67 \text{ km/minute}$$

2. Because there are 60 minutes in an hour, convert this to kilometers per hour using a second conversion factor:

$$\frac{1.67 \text{ km}}{1 \text{ minute}} \times \frac{60 \text{ minute}}{1 \text{ hour}} = 100 \text{ km/hour}$$

17 How many hours are in 1 week?

18 How many hours are in 1 year?

Counting Significant Figures

You may plug numbers into your calculator and come up with an answer like 1.532984529045, but that number isn't likely to please your instructor. Why? Because in physics problems, you use significant digits to express your answers. *Significant digits*, also often called *significant figures*, represent the accuracy with which you know your values.

For example, if you know only the values you're working with to two significant digits, your answer should be 1.5, which has two significant digits, not 1.532984529045, which has 13! Here's how it works: Suppose you're told that a skater traveled 10.0 meters in 7.0 seconds. Note the number of digits: The first value has three significant figures, the other only two. The rule is that when you multiply or divide numbers, the result has the number of significant digits that equals the smallest number of significant digits in any of the original numbers. So if you want to figure out how fast the skater was going, you divide 10.0 by 7.0, and the result should have only two significant digits — 1.4 meters per second.

On the other hand, when you're adding or subtracting numbers, the rule is that the last significant digit in the result corresponds to the last significant digit in the least accurate measurement. How does that work? Take a look at this addition example:

$$\begin{array}{r} 5.1 \\ 12 \\ + 7.73 \\ \hline 24.83 \end{array}$$

So is the result 24.83? No, it's not. The 12 has no significant digits to the right of the decimal point, so the answer shouldn't have any either. That means you should round the value of the result up to 25.



REMEMBER

Zeros used just to fill out values down to (or up to) the decimal point aren't considered significant. For example, the number 3,600 has only two significant digits by default. That's not true if the value was actually measured to be 3,600, of course, in which case it's usually expressed as 3,600; the final decimal indicates that all the digits are significant.

Rounding numbers in physics usually works the same way as it does in math: When you want to round to three places, for example, and the number in the fourth place is a five or greater, you add one to the third place (and ignore or replace with zeros any following digits).



EXAMPLE

Q. You're multiplying 12.01 by 9.7. What should your answer be, keeping in mind that you should express it in significant digits?

A. The correct answer is 120.

1. The calculator says the product is 116.497.
2. The number of significant digits in your result is the same as the smallest number of significant digits in any of the values being multiplied. That's two here (because of 9.7), so your answer rounds up to 120.

19 What is 19.3 multiplied by 26.12, taking into account significant digits?

20 What is the sum of 7.9, 19, and 5.654, taking into account significant digits?

Coming Prepared with Some Algebra

It's a fact of life: You need to be able to do algebra to handle physics problems. Take the following equation, for example, which relates the distance something has traveled (s) to its acceleration and the time it has been accelerated:

$$s = \frac{1}{2}at^2$$

Now suppose that the physics problem asks you for the acceleration, not the distance. You have to rearrange things a little here to solve for the acceleration. So when you multiply both sides by 2 and divide both sides by t^2 , here's what you get:

$$\frac{2}{t^2} \cdot s = \frac{2}{t^2} \cdot \frac{1}{2}at^2$$

Cancelling out and swapping sides, you solve for a like this:

$$a = \frac{2s}{t^2}$$

So that's putting a little algebra to work. All you had to do was move variables around the equation to get what you wanted. The same approach works when solving physics problems (most of the time). On the other hand, what if you had to solve the same problem for the time, t ? You would do that by rearranging the variables like so:

$$t = \sqrt{2s/a}$$

The lesson in this example is that you can extract all three variables — distance, acceleration, and time — from the original equation. Should you memorize all three versions of this equation? Of course not. You can just memorize the first version and use a little algebra to get the rest.

The following practice questions call on your algebra skills:



EXAMPLE

Q. The equation for final speed, v_f — where the initial speed is v_o , the acceleration is a , and the time is t — is $v_f = v_o + at$. Solve for acceleration.

A. The correct answer is $a = (v_f - v_o) / t$

To solve for a , divide both sides of the equation by time, t .

21 The equation for potential energy, PE , for a mass m at height h , where the acceleration due to gravity is g , is $PE = mgh$. Solve for h .

22 The equation relating final speed, v_f , to original speed, v_o , in terms of acceleration a and distance s is $v_f^2 = v_o^2 + 2as$. Solve for s .

23 The equation relating distance s to acceleration a , time t , and speed v is $s = v_o t + \frac{1}{2}at^2$. Solve for v_o .

24 The equation for kinetic energy is $KE = \frac{1}{2}mv^2$. Solve for v .

Being Prepared with Trigonometry

Physics problems also require you to have some trigonometry under your belt. To see what kind of trig you need, take a look at Figure 1-1, which shows a right triangle. The long side is called the *hypotenuse*, and the angle between x and y is 90° .

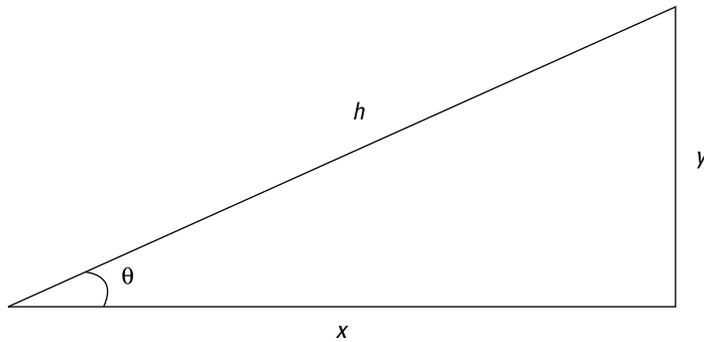


FIGURE 1-1:
A triangle.

© John Wiley & Sons, Inc.

Physics problems require you to be able to work with sines, cosines, and tangents. Here's what they look like for Figure 1-1:

$$\sin \theta = y / h$$

$$\cos \theta = x / h$$

$$\tan \theta = y / x$$

You can find the length of one side of the triangle if you're given another side and an angle (not including the right angle). Here's how to relate the sides:

$$x = h \cos \theta = y / \tan \theta$$

$$y = h \sin \theta = x \tan \theta$$

$$h = y / \sin \theta = x / \cos \theta$$

And here's one more equation, the Pythagorean theorem. It gives you the length of the hypotenuse when you plug in the other two sides:

$$h = \sqrt{x^2 + y^2}$$

25 Given the hypotenuse h and the angle θ , what is the length x equal to?

26 If $x = 3$ m and $y = 4$ m, what is the length of h ?

Answers to Problems about Physics Basics

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 **grams**

The unit of mass in the CGS system is the gram.

2 **seconds**

The unit of time in the MKS system is the second.

3 **newtons**

The unit of force in the MKS system is the newton.

4 **ergs**

The unit of energy in the CGS system is the erg.

5 4.3×10^{-3}

You have to move the decimal point three places to the right.

6 4.3×10^5

You have to move the decimal point five places to the left.

7 5.6×10^{-7}

You have to move the decimal point seven places to the right.

8 6.7×10^3

You have to move the decimal point three places to the left.

9 **235 cm**

Convert 2.35 meters into centimeters:

$$2.35 \text{ m} \times \frac{100 \text{ cm}}{1 \text{ m}} = 235 \text{ cm}$$

10 **75 sec**

Convert 1.25 minutes into seconds:

$$1.25 \text{ min} \times \frac{60 \text{ sec}}{1 \text{ min}} = 75 \text{ sec}$$

11 **79 in**

Convert 2.0 meters into inches:

$$2.0 \text{ m} \times \frac{39.37 \text{ in}}{1 \text{ m}} = 79 \text{ in}$$

12 **3,250 g**

Convert 3.25 kilograms into grams:

$$3.25 \text{ kg} \times \frac{1,000 \text{ g}}{1 \text{ kg}} = 3,250 \text{ g}$$

13 **91.4 cm**

1. 1 yard is 3 feet, so convert that to inches:

$$3 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} = 36 \text{ in}$$

2. Use a second conversion factor to convert that into centimeters:

$$36 \text{ in} \times \frac{2.54 \text{ cm}}{1 \text{ in}} = 91.4 \text{ cm}$$

14 **1.0×10^5 cm**

1. Convert 1 kilometer to meters:

$$1 \text{ km} \times \frac{1,000 \text{ m}}{1 \text{ km}} = 1.0 \times 10^3 \text{ m}$$

2. Use a second conversion factor to convert that into centimeters:

$$1 \times 10^3 \text{ m} \times \frac{100 \text{ cm}}{1 \text{ m}} = 1.0 \times 10^5 \text{ cm}$$

15 **4.0×10^{-9} in**

1. Convert 1 angstrom to centimeters:

$$1 \text{ \AA} \times \frac{10^{-8} \text{ cm}}{1 \text{ \AA}} = 10^{-8} \text{ cm}$$

2. Use a second conversion factor to convert that into inches:

$$10^{-8} \text{ cm} \times \frac{1 \text{ in}}{2.54 \text{ cm}} = 4.0 \times 10^{-9} \text{ in}$$

16 **120 in**

1. Convert 3.0 meters into centimeters:

$$3.0 \text{ m} \times \frac{100 \text{ cm}}{1 \text{ m}} = 300 \text{ cm}$$

2. Use a second conversion factor to convert that into inches:

$$300 \text{ cm} \times \frac{1 \text{ in}}{2.54 \text{ cm}} = 120 \text{ in}$$

17 **168 hours**

1. Convert 1 week into days:

$$1 \cancel{\text{ week}} \times \frac{7 \text{ days}}{1 \cancel{\text{ week}}} = 7 \text{ days}$$

2. Use a second conversion factor to convert that into hours:

$$7 \cancel{\text{ days}} \times \frac{24 \text{ hours}}{1 \cancel{\text{ day}}} = 168 \text{ hours}$$

18 **8,760 hours**

1. Convert 1 year into days:

$$1 \cancel{\text{ year}} \times \frac{365 \text{ days}}{1 \cancel{\text{ year}}} = 365 \text{ days}$$

2. Use a second conversion factor to convert that into hours:

$$365 \cancel{\text{ days}} \times \frac{24 \text{ hours}}{1 \cancel{\text{ day}}} = 8,760 \text{ hours}$$

19 **504**

1. The calculator says the product is 504.116.
2. 19.3 has three significant digits, and 26.12 has four, so you use three significant digits in your answer. That makes the answer 504.

20 **33**

1. Here's how you do the sum:

$$\begin{array}{r} 7.9 \\ +19 \\ + 5.654 \\ \hline 32.554 \end{array}$$

2. The value 19 has no significant digits after the decimal place, so the answer shouldn't either, making it 33 (32.554 rounded up).

21 **$h = PE / mg$**

Divide both sides by mg to get your answer.

22 **$s = \frac{v_f^2 - v_o^2}{2a}$**

Divide both sides by $2a$ to get your answer.

$$(23) v_o = \frac{s}{t} - \frac{1}{2}at$$

1. Subtract $\frac{1}{2}at^2$ from both sides:

$$s - \frac{1}{2}at^2 = v_o t$$

2. Divide both sides by t to get your answer.

$$(24) v = \sqrt{\frac{2KE}{m}}$$

1. Multiply both sides by $2/m$:

$$\frac{2}{m}KE = v^2$$

2. Take the square root to get your answer.

$$(25) x = h \cos \theta$$

Your answer comes from the definition of cosine.

$$(26) 5 \text{ m}$$

1. Start with the Pythagorean theorem:

$$h = \sqrt{x^2 + y^2}$$

2. Plug in the numbers, and work out the answer:

$$h = \sqrt{(3 \text{ m})^2 + (4 \text{ m})^2} = \sqrt{9 \text{ m}^2 + 16 \text{ m}^2} = \sqrt{25 \text{ m}^2} = 5 \text{ m}$$

- » Thinking about displacement
- » Checking out velocity
- » Remembering acceleration

Chapter 2

The Big Three: Displacement, Velocity, and Acceleration

Being able to connect displacement, velocity, and acceleration is fundamental to working with physics. These things concern people every day, and physics has made an organized study of them.

Problems that connect displacement, velocity, and acceleration are all about understanding movement, and that's the topic of this chapter — putting numbers into the discussion. You'll often find physics problems about cars starting and stopping, horses racing, and rocket ships zooming back and forth. And after you finish this chapter, you'll be a real pro at solving them.

From Point A to B: Displacement

Displacement occurs when something moves from here to there. For example, suppose that you have a ball at the zero position, as in Figure 2-1a.

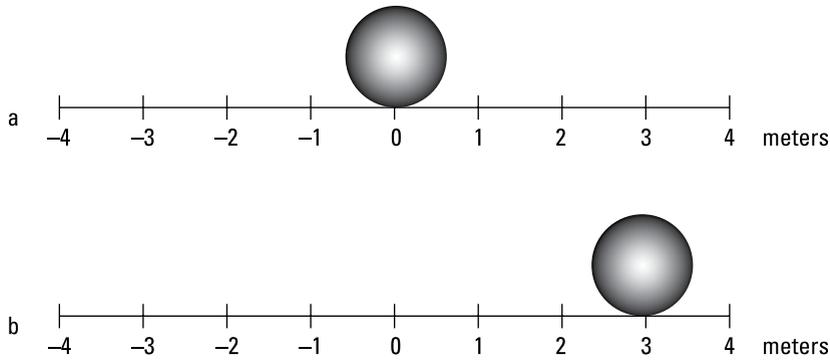


FIGURE 2-1:
A moving ball.

© John Wiley & Sons, Inc.

Now suppose that the ball rolls over to a new point, 3 meters to the right, as you see in Figure 2-1b. The ball is at a new location, so there's been displacement. In this case, the displacement is just 3 meters to the right. In physics terms, you'll often see displacement referred to as the variable s . In this case, $s = +3$ meters.

Like any other measurement in physics, displacement is always expressed in units, usually centimeters or meters, as in this example. Of course, you also can use kilometers, inches, feet, miles, or even *light years* (the distance light travels in one year — 5,878,500,000,000 miles).

In addition to a distance, a displacement also includes a direction. For example, a displacement of 12 miles north is very different from a displacement of 12 miles south.

The following example question focuses on displacement.



EXAMPLE

Q. You've taken the pioneers' advice to "Go West." You started in New York City and went west 10 miles the first day, 14 miles west the next day, and then back east 9 miles on the third day. What is your displacement from New York City after three days?

A. $s = 15$ miles west of New York City

1. You first went west 10 miles, so at the end of the first day, your displacement was 10 miles west.
2. Next, you went west 14 miles, putting your displacement at 10 miles + 14 miles = 24 miles west of New York City.
3. Finally, you traveled 9 miles east, leaving you at 24 miles – 9 miles = 15 miles west of New York City. So $s = 15$ miles west of New York City.

1 Suppose that the ball in Figure 2-1b now moves 1 more meter to the right. What is its new displacement from the origin, 0?

2 Suppose that the ball in Figure 2-1, which started 4.0 meters to the right of the origin, moves 6.0 meters to the left. What is its new displacement from the origin — in inches?

Reading That Speedometer

In physics terms, what is speed? It's the same as the conventional idea of speed: *Speed* is distance divided by time, which is what a speedometer measures. The related term *velocity* refers to a speed with an associated direction. In other words, velocity is displacement (distance in a particular direction) divided by time. To measure velocity, you might use a speedometer in combination with a compass.

For example, if you went a displacement s in a time t , then your velocity, v , is determined as follows:

$$v = \frac{s}{t}$$

Technically speaking, velocity is the change in position divided by the change in time, so you also can represent it like this if, for example, you're moving along the x axis:

$$v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_o}{t_f - t_o}$$



EXAMPLE

Q. Suppose that you want to drive from New York City to Los Angeles to visit your uncle's family, a distance of about 2,781 miles. The trip takes you four days. What was your speed in miles per hour?

A. $v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_o}{t_f - t_o} = 28.97 \text{ miles/hour}$

1. Start by figuring out your speed (the distance traveled divided by the time taken to travel that distance):

$$\frac{2,781 \text{ miles}}{4 \text{ days}} = 695.25$$

2. Okay, the speed is 695.25, but 695.25 *what*? This solution divides miles by days, so it's 695.25 miles per day — not exactly a standard unit of measurement. So what is that in miles per hour? To determine that, you cancel “days” out of this equation and put in “hours.” Because 24 hours are in a day, you can multiply as follows (note that “days” cancel out, leaving miles over hours, or miles per hour):

$$\frac{695.25 \text{ miles}}{1 \text{ day}} \times \frac{1 \text{ day}}{24 \text{ hours}} = 28.97 \text{ miles/hour}$$

So your speed was 28.97 miles per hour. That's your average speed, averaged over both day and night.

3 Suppose that you used your new SpeedPass to get you through the tollbooths at both ends of your trip, which was 90.0 miles on the turnpike and took you 1 hour and 15 minutes. On your return home, you're surprised to find a traffic ticket for speeding in the mail. How fast did you go, on average, between the tollbooths? Was the turnpike authority justified in sending you a ticket, given that the speed limit was 65 mph?

4 Suppose that you and a friend are determined to find out who drives faster. You both start your trips in Chicago. Driving nonstop, you reach Los Angeles — a distance of 2,018 miles — in 1.29 days, and your friend, also driving nonstop, reaches Miami — a distance of 1,380 miles — in 0.89 days. Who drove faster?

Putting Pedal to Metal: Acceleration

In physics terms, *acceleration* is the amount by which your velocity changes in a given amount of time. In terms of equations, it works like this:

$$a = \frac{\Delta v}{\Delta t}$$

Given initial and final velocities, v_o and v_f , and initial and final times over which your speed changed, t_o and t_f , you can also write the equation like this:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o}$$

To get the units of acceleration, you divide velocity by time as follows:

$$a = \frac{v_f - v_o}{t_f - t_o} = \frac{\text{displacement/time}}{\text{time}} = \frac{\text{displacement}}{\text{time}^2}$$

Displacement over time squared? Don't let that throw you. You end up with time squared in the denominator just because its velocity divided by time — that's something you get used to when solving physics problems. In other words, acceleration is the *rate* at which your velocity or speed changes because rates have time in the denominator.

So for acceleration, you can expect to see units of meters per second², or centimeters per second², or miles per second², or feet per second², or even kilometers per hour².

Like velocity, acceleration has a magnitude and direction. For example, a car might be traveling at +25 m/s north as it reaches a red stop light. An acceleration of -5 m/s^2 (that is, an acceleration toward the south) may bring the car safely to a stop at the light. An acceleration of +5 m/s² will likely result in an accident!



EXAMPLE

Q. Suppose you're driving 90.0 kilometers an hour and suddenly see red flashing lights in the rearview mirror. "Great," you think, and you pull over, taking 20.0 seconds to come to a stop. How fast did you decelerate, in m/sec²?

A. -1.25 m/sec^2

1. First convert to kilometers per second:

$$\frac{90 \text{ kilometers}}{\text{hour}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = 0.0250 \text{ kilometers per second}$$

2. Convert from kilometers per second to meters per second:

$$\frac{0.0250 \text{ kilometers}}{\text{second}} \times \frac{1,000 \text{ meters}}{1 \text{ kilometer}} = \frac{25.0 \text{ meters}}{\text{second}}$$

3. What was your acceleration? That calculation looks like this:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o} = \frac{0 - 25.0 \text{ m/second}}{20.0 \text{ seconds}} = -1.25 \text{ m/s}^2$$

In other words, -1.25 m/s^2 , *not* $+1.25 \text{ m/s}^2$. There's a big difference between positive and negative in terms of solving physics problems — and in terms of law enforcement. If you accelerated at $+1.25 \text{ m/s}^2$ instead of accelerating at -1.25 m/s^2 , you'd end up going 180 kilometers per hour at the end of 20.0 seconds, not 0 kilometers per hour.



REMEMBER

The *sign* of the acceleration tells you *how* the velocity is changing. A positive acceleration means the velocity is increasing in the positive direction, and a negative acceleration (also known as *deceleration*) tells you the velocity is increasing in the negative direction.

-
- | | |
|--|---|
| <p>5 A rocket ship is going to land on the moon in exactly 2.0 hours. There's only one problem: It's going 17,000 miles an hour. What does its deceleration need to be, in miles per second², to land on the moon safely at 0.0 miles per hour?</p> | <p>6 You're stopped at a red light when you see a monster SUV careening toward you. In a lightning calculation, you determine you have 0.8 seconds before it hits you and you must be going at least 1.0 miles an hour forward at that time to avoid the SUV. What must your acceleration be, in miles per hour²? Can you avoid the SUV?</p> |
| <p>7 A bullet comes to rest in a block of wood in 1.0×10^{-2} seconds, with an acceleration of $-8.0 \times 10^4 \text{ m/s}^2$. What was its original speed, in meters per second?</p> | <p>8 The light turns red, and you ease to a halt. Checking your stopwatch, you see that you stopped in 4.5 seconds. Your deceleration was 1.23×10^{-3} miles per second². What was your original speed in miles per hour?</p> |

Connecting Acceleration, Time, and Displacement

You know that you can relate velocity with displacement and time. And you know that you can relate velocity and time to get acceleration. You also can relate displacement with acceleration and time:

$$s = \frac{1}{2}a(t_f - t_o)^2$$

If you don't start off at zero velocity, you use this equation:

$$s = v_o(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2$$



EXAMPLE

- Q.** You climb into your drag racer, waving nonchalantly at the cheering crowd. You look down the quarter-mile track, and suddenly the flag goes down. You're off, getting a tremendous kick from behind as the car accelerates quickly. A brief 5.5 seconds later, you pass the end of the course and pop the chute.

You know the distance you went: 0.25 miles, or about 402 meters. And you know the time it took: 5.5 seconds. So just how hard was the kick you got — the acceleration — when you blasted down the track?

- A.** 26.6 m/s²

1. You know

$$s = \frac{1}{2}at^2$$

You can rearrange this equation with a little algebra (just divide both sides by t^2 and multiply by 2) to get

$$a = \frac{2s}{t^2}$$

2. Plugging in the numbers, you get

$$a = \frac{2s}{t^2} = \frac{2(402 \text{ meters})}{(5.5 \text{ seconds})^2} = 26.6 \text{ meters/second}^2$$

What's 26.6 meters/second² in more understandable terms? The acceleration due to gravity, g , is 9.8 meters/second², so this is about 2.7 g . And that's quite a kick.

9 The light turns green, and you accelerate at 10.0 m/s^2 . After 5.00 seconds, how far have you traveled?

10 A stone drops under the influence of gravity, 9.8 m/s^2 . How far does it drop in 1.2 seconds?

11 A car is going 60.0 miles per hour and accelerating at $10.0 \text{ miles per hour}^2$. How far does it go in 1.0 hour?

12 A motorcycle is going 60 miles per hour, and decelerating at $60 \text{ miles per hour}^2$. How far does it go in 1 hour?

13 An eagle starts at a speed of 50.0 m/s and, decelerating at 10.0 m/s^2 , comes to rest on a peak 5.00 seconds later. How far is the peak from the eagle's original position?

14 A trailer breaks loose from its truck on a steep incline. If the truck was moving uphill at 20.0 m/s when the trailer broke loose, and the trailer accelerates down the hill at 10.0 m/s^2 , how far downhill does the trailer go after 10.0 seconds?

- 15 A block of wood is shooting down a track at 10.0 m/s and is slowing down because of friction. If it comes to rest in 20.0 seconds, what is its deceleration, in meters per second²?

- 16 A minivan puts on the brakes and comes to a stop in 3.0 seconds. If the deceleration is 4.0 m/s², how far does the minivan travel, in meters, before stopping?

Connecting Velocity, Acceleration, and Displacement

Suppose you have a drag racer whose acceleration is 26.6 m/s² with a final speed of 146.3 m/s. What is the total distance traveled?

This scenario sets you up to use a handy motion equation:

$$v_f^2 - v_o^2 = 2as = 2a(x_f - x_o)$$

This is the equation you use to relate velocity, acceleration, and distance.



- Q.** A drag racer's acceleration is 26.6 m/s², and at the end of the race, the final speed is 146.3 m/s. What is the total distance the drag racer traveled?

EXAMPLE

- A.** 402 meters

1. To solve this problem, you need to relate speed, acceleration, and distance, so you start with this equation:

$$v_f^2 - v_o^2 = 2as = 2a(x_f - x_o)$$

2. In this scenario, v_o is 0, which makes this equation simpler:

$$v_f^2 = 2as$$

3. Solve for s :

$$s = \frac{1}{2a} v_f^2$$

4. Plug in the numbers:

$$s = \frac{1}{2a} v_f^2 = \frac{1}{2(26.6 \text{ m/s}^2)} (146.3 \text{ m/s})^2 = 402 \text{ meters}$$

So the answer is 402 meters, about a quarter of a mile — standard for a drag racing track.

17 A bullet is accelerated over a meter-long rifle barrel at an acceleration of $400,000 \text{ m/s}^2$. What is its final speed?

18 A dragster starts from rest and is accelerated at 5.0 m/s^2 . What is its speed 5.0×10^2 meters later?

19 A rocket is launched with an acceleration of 100.0 m/s^2 . After 100.0 kilometers, what is its speed in meters per second?

20 A motorcycle is going 40.0 m/s and is accelerated at 6.00 m/s^2 . What is its speed after 2.00×10^2 meters?

Answers to Problems about Displacement, Velocity, and Acceleration

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 $s = +4$ meters

The ball was originally at +3 meters and moved 1 meter to the right. $+3 + 1 = +4$ meters.

2 $s = -80$ inches

1. The ball started at 4 meters and moved 6 meters to the left, putting it at $+4.0 - 6.0 = -2.0$ meters with respect to the origin.

2. Convert -2.0 meters into inches:

$$-2.0 \cancel{\text{meters}} \times \frac{39.37 \text{ inches}}{1 \cancel{\text{meter}}} = -80 \text{ inches}$$

3 $v = 72$ miles an hour. The ticket was justified.

1. It took you 1 hour and 15 minutes, or 1.25 hours, to travel 90.0 miles.

2. Divide 90.0 miles by 1.25 hours:

$$\frac{90.0 \text{ miles}}{1.25 \text{ hours}} = 72.0 \text{ miles/hour}$$

This is faster than the speed limit of 65 miles/hour.

4 **Your speed = 1,564 miles per day; your friend's speed = 1,551 miles per day. You're faster.**

1. Note that to simply compare speeds, there's no need to convert to miles per hour — miles per day will do fine. First, calculate your speed:

$$\frac{2,018 \text{ miles}}{1.29 \text{ days}} = 1,564 \text{ miles/day}$$

2. Next, calculate your friend's speed:

$$\frac{1,380 \text{ miles}}{0.89 \text{ days}} = 1,551 \text{ miles/day}$$

So you were faster than your friend.

5 6.6×10^{-4} miles per second²

1. Start by converting 17,000 miles per hour into miles per second:

$$\frac{17,000 \text{ miles}}{\text{hour}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = 4.72 \text{ miles per second}$$

2. To land on the moon, v_f must be 0 miles per second, and $t_f - t_o = 2.0$ hours, or $2.0 \times 3,600$ seconds = 7,200 seconds, so:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o} = \frac{0 - 4.72 \text{ miles per second}}{7,200 \text{ seconds}}$$

3. Calculating this yields

$$\frac{-4.72}{7,200} = 0.00066 = -6.6 \times 10^{-4} \text{ miles per second}^2$$

So the rocket needs a constant deceleration of 6.6×10^{-4} miles per second² to land on the moon at a speed of 0 miles per second, touching down lightly.

6 4.5×10^3 miles per hour². You will avoid the collision.

1. Start by converting 0.8 seconds into hours to get all the quantities in units you want — miles and hours:

$$0.8 \text{ seconds} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = 2.22 \times 10^{-4} \text{ hours}$$

2. Calculate the acceleration needed to get you to 1.0 miles per hour:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o} = \frac{1.0 \text{ miles per hour} - 0}{2.22 \times 10^{-4} \text{ hour} - 0} = 4.5 \times 10^3 \text{ miles per hour}^2$$

7 800 m/s

1. You can calculate the change in speed, because it's acceleration multiplied by time:

$$\Delta v = a(\Delta t) = (-8.0 \times 10^4 \text{ m/s}^2)(1.0 \times 10^{-2} \text{ s}) = -8.0 \times 10^2 \text{ m/s}$$

2. So if the bullet lost 800 m/s of speed to come to a rest ($v = 0$), it must have been going 800 m/s originally.

8 20 miles per hour

1. The change in speed is acceleration multiplied by time, so:

$$\Delta v = a(\Delta t) = (1.23 \times 10^{-3} \text{ miles per second}^2)(4.5 \text{ s}) = 5.55 \times 10^{-3} \text{ miles per second}$$

2. Convert this result to miles per hour:

$$\frac{5.55 \times 10^{-3} \text{ miles}}{\text{second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = 20 \text{ miles per hour}$$

9 125 meters

1. You want to relate distance to acceleration and time, so use this equation:

$$s = \frac{1}{2}at^2$$

2. Plug in the numbers:

$$s = \frac{1}{2}at^2 = \frac{1}{2}(10.0 \text{ m/s}^2)(5.00 \text{ s})^2 = 125 \text{ meters}$$

10 7.06 meters (that's 7.06 meters downward)

1. To relate distance to acceleration and time, you use this equation:

$$s = \frac{1}{2}at^2$$

2. Substitute the numbers:

$$s = \frac{1}{2}at^2 = \frac{1}{2}(-9.8 \text{ m/s}^2)(1.2 \text{ s})^2 = -7.06 \text{ meters}$$

11 65 miles

1. To relate distance to speed, acceleration, and time, you use this equation:

$$s = v_o(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2$$

2. Plug in the numbers:

$$\begin{aligned} s &= v_o(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2 \\ &= (60.0 \text{ miles/hour})(1.0 \text{ hour}) + \frac{1}{2}(10.0 \text{ miles/hour}^2)(1.0 \text{ hour})^2 \\ &= 65 \text{ miles} \end{aligned}$$

12 30 miles

1. You want to relate distance to speed, acceleration, and time, so you use this equation:

$$s = v_o(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2$$

2. Plug in the numbers:

$$\begin{aligned} s &= v_o(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2 \\ &= (60 \text{ miles/hour})(1.0 \text{ hour}) - \frac{1}{2}(60 \text{ miles/hour}^2)(1.0 \text{ hour})^2 \\ &= 30 \text{ miles} \end{aligned}$$

13 125 meters

1. To connect distance with speed, acceleration, and time, you use this equation:

$$s = v_o(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2$$

2. Plug in the numbers:

$$\begin{aligned} s &= v_o(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2 \\ &= (50.0 \text{ m/s})(5.00 \text{ s}) - \frac{1}{2}(10.0 \text{ m/s}^2)(5.00 \text{ s})^2 \\ &= 125 \text{ meters} \end{aligned}$$

14 -300 meters

1. To relate acceleration to speed and time, use this equation:

$$s = v_o(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2$$

2. Plug in the numbers:

$$\begin{aligned} s &= v_o(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2 \\ &= (20.0 \text{ m/s})(10.0 \text{ s}) - \frac{1}{2}(10.0 \text{ m/s}^2)(10.0 \text{ s})^2 \\ &= -300 \text{ meters} \end{aligned}$$

15 $-5.00 \times 10^{-1} \text{ m/s}^2$

1. To relate acceleration to speed and time, use this equation:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o}$$

2. Plug in the numbers:

$$\begin{aligned} a &= \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o} \\ &= \frac{0 - 10.0 \text{ m/s}}{20.0 \text{ s}} \\ &= -0.500 \text{ m/s}^2 = -5.00 \times 10^{-1} \text{ m/s}^2 \end{aligned}$$

16 18 meters

1. To relate acceleration to velocity and time, use this equation:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o}$$

2. Solve for v_o :

$$v_o = v_f - a(t_f - t_o) = 0 - (-4.0 \text{ m/s}^2)(3.0 \text{ s}) = 12 \text{ m/s}$$

3. Plug in the numbers:

$$\begin{aligned} s &= v_o(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2 \\ &= (12 \text{ m/s})(3.0 \text{ s}) - \frac{1}{2}(4.0 \text{ m/s}^2)(3.0 \text{ s})^2 \\ &= 18 \text{ meters} \end{aligned}$$

17 $v_f = 900 \text{ m/s}$

1. Start with this equation:

$$v_f^2 - v_o^2 = 2as = 2a(x_f - x_o)$$

2. v_o is 0, so that makes things easier. Plug in the numbers:

$$\begin{aligned}v_f^2 &= 2as \\ &= 2(400,000 \text{ m/s}^2)1.0 \text{ m} \\ &= 800,000(\text{meters per second})^2\end{aligned}$$

3. Take the square root:

$v_f = 894$ meters per second, which rounds to 900 with significant figures

18 $v_f = 71 \text{ m / s}$

1. You want to find speed in terms of distance and acceleration, so use this equation:

$$v_f^2 - v_o^2 = 2as = 2a(x_f - x_o)$$

2. Plug in the numbers:

$$\begin{aligned}v_f^2 &= 2as \\ &= 2(5.0 \text{ m/s}^2)500 \text{ m} \\ &= 5,000(\text{meters per second})^2\end{aligned}$$

3. Take the square root:

$v_f = 70.7$ meters per second, which rounds to 71 with significant figures

19 $v_f = 4,472 \text{ m / s}$

1. You want to find the speed of the rocket, having been given distance and acceleration, so use this equation:

$$v_f^2 - v_o^2 = 2as = 2a(x_f - x_o)$$

2. 100 kilometers is 100,000 meters, so plug in the numbers:

$$\begin{aligned}v_f^2 &= 2as \\ &= 2(100.0 \text{ m/s}^2)100,000 \text{ m} \\ &= 2.000 \times 10^7 (\text{meters per second})^2\end{aligned}$$

3. Take the square root to get the rocket's speed:

$v_f = 4,472$ meters per second

20 $v_f = 63.2 \text{ m/s}$

1. To determine the motorcycle's final speed, use this equation:

$$v_f^2 - v_o^2 = 2as$$

2. $v_o = 40$ meters per second, so plug in the numbers:

$$\begin{aligned}v_f^2 - (40 \text{ m/s})^2 &= 2as \\ &= 2(6.00 \text{ m/s}^2)200 \text{ m} \\ &= 2,400(\text{meters per second})^2\end{aligned}$$

3. That means that v_f^2 is

$$v_f^2 = 4,000 (\text{meters per second})^2$$

4. Take the square root to get v_f :

$$v_f = 63.2 \text{ meters per second}$$

- » Understanding what makes a vector
- » Expressing vectors in different forms
- » Converting vectors to different forms
- » Adding vectors
- » Expressing velocity as vectors

Chapter 3

Vectors: Knowing Where You're Headed

Although an object in motion has a speed, it also has a direction. Together, the speed and the direction describe the object's motion. For example, you may be heading off to your grandmother's house at 60 miles per hour, but unless you're pointed in the right direction, you're not going to get there. As discussed in Chapter 2, speed associated with a direction is called *velocity*.

Physics takes note of the fact that objects in motion need two quantities to fully describe that motion — speed and direction — by saying that such motion is a *vector* quantity. Velocity, then, is an example of a vector quantity.

Creating a Vector

A vector is a combination of exactly two values: a magnitude (like the speed of an object in motion) and a direction (such as the direction of an object in motion). All kinds of things can be described with vectors, including velocity, acceleration, displacement, magnetic fields, electric fields, and many more.

Vectors are defined by a *magnitude* (the length of the vector) and a direction. For example, take a look at the vector in Figure 3-1.

FIGURE 3-1: **A**
A vector.

© John Wiley & Sons, Inc.

In physics, vectors are written in bold type. The vector in Figure 3-1 — I'll call it **A** — represents the displacement of a golf ball from the tee. Its length is 100 yards, and its direction is 15° north of due east. That's all you need to have a vector — a magnitude and a direction.

Now take a look at the two vectors in Figure 3-2, **A** and **B**. These two vectors are considered equal, which is written as $\mathbf{A} = \mathbf{B}$.

FIGURE 3-2:
Two vectors.

© John Wiley & Sons, Inc.



REMEMBER

Two vectors are considered equal if they have the same magnitude and direction. They do *not* need to start at the same point. The magnitude of a vector **A** — that is, its length — is written as **A**, not in bold type.

Figure 3-3 shows the standard coordinate system for vectors. Note the *x* and *y* axes, which vectors are measured against.

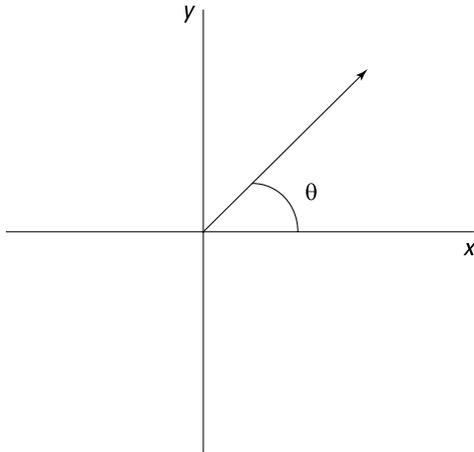


FIGURE 3-3:
Vector
coordinate
system.

© John Wiley & Sons, Inc.

The *x* and *y* axes are measured using some standard physics units, such as centimeters. Positive *x* is to the right, negative *x* is to the left; positive *y* is up, negative *y* is down. The center of the graph, where the axes meet, is called the *origin*. A vector is commonly described by its length and its angle from the positive *x* axis (0° to 360°).



Q. Suppose the vector in Figure 3-3 is 3.0 centimeters long and at an angle of 45° with respect to the x axis. How would you exactly describe this vector?

A. The correct answer is 3.0 centimeters long and at an angle of $+45^\circ$ with respect to the positive x axis.

1 A marble starts at the origin and rolls 45 meters to the right. Describe where it ends up, in vector notation.

2 A marble starts at the origin and rolls 45 meters to the right. Then it moves 90 meters to the left. Describe where it ends up, in vector notation.

Understanding Vector Components

In addition to specifying a vector with a magnitude and a direction, you can specify it with a pair of coordinates as measured from the origin. For example, take a look at the vector in Figure 3-4, where the measurements are in centimeters.

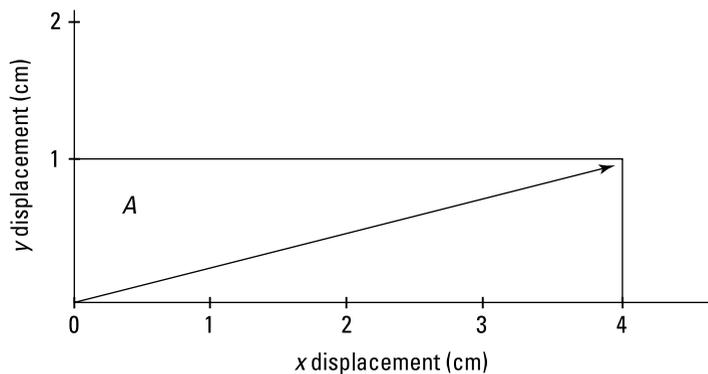


FIGURE 3-4: Resolving a vector.

© John Wiley & Sons, Inc.

You can describe the vector in Figure 3-4 with a length and an angle, of course, but you also can describe it by the coordinates of the tip of its arrow. In this case, that tip is at 4 centimeters to the right and 1 centimeter up from the origin. You notate that location as (4, 1) cm, which is a valid way of expressing a vector.

So the two ways of expressing a vector are

- » As a magnitude and a direction; for example, **A** is a vector along the x axis of length 5 meters
- » As a pair of coordinates (or vector components) corresponding to the tip of the vector (assuming the tail of the vector is at the origin); for example, **A** = (5, 0) m



EXAMPLE

Q. Suppose you walk 3.0 meters to the right of the origin. What is your displacement vector in terms of vector components?

A. The correct answer is (3.0, 0) m.

Your x coordinate in meters is 3.0, and your y coordinate is 0, so your vector displacement is (3.0, 0) m.

3 A marble starts at the origin and moves to the right 5.0 centimeters. What is its new displacement in vector component terms?

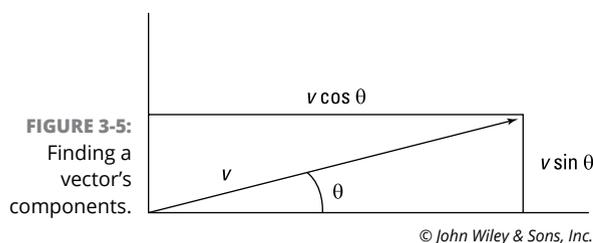
4 Suppose you move to the right of the origin by 3.5 meters and then up 5.6 meters. What is your final vector from the origin, in vector component terms?

Finding a Vector's Components

You can convert from the magnitude/angle way of specifying a vector to the coordinate way of expression. Doing so is essential for the kinds of operations you can expect to execute on vectors, such as when adding vectors.

For example, you have one vector at 15° and one at 19° , and you want to add them together. How the heck do you do that? If you were to convert them into their coordinates, (a, b) and (c, d) , the answer would be trivial because you only have to add the x and y coordinates to get the answer: $(a + c, b + d)$.

To see how to convert between the two ways of looking at vectors, take a look at vector \mathbf{v} in Figure 3-5. The vector can be described as having a magnitude v at an angle of θ .



To convert this vector into the coordinate way of looking at vectors, you have to use the trigonometry shown in the figure. The x coordinate equals $v \cos \theta$, and the y coordinate equals $v \sin \theta$:

$$v_x = v \cos \theta$$

$$v_y = v \sin \theta$$

Keep these relationships in mind because you'll come across them often in physics questions.



EXAMPLE

- Q.** Suppose that you've walked away from the origin so you're now 5.0 kilometers from the origin, at an angle of 45° . Resolve that into vector coordinates.

- A.** The correct answer is $(3.5, 3.5)$ km.
1. Apply the equation $v_x = v \cos \theta$ to find the x coordinate. That's $5.0 \text{ km} \cos 45^\circ$, or 3.5 km.
 2. Apply the equation $v_y = v \sin \theta$ to find the y coordinate. That's $5.0 \text{ km} \sin 45^\circ$, or 3.5 km.

5 Resolve a vector 3.0 meters long at 15° into its components.

6 Resolve a vector 9.0 meters long at 35° into its components.

7 Resolve a vector 6.0 meters long at 125° into its components.

8 Resolve a vector 4.0 meters long at 255° into its components.

Finding a Vector's Magnitude and Direction

If you're given the components of a vector, such as (3, 4), you can convert it easily to the magnitude/angle way of expressing vectors using trigonometry.

For example, take a look at the vector in Figure 3-5. Suppose that you're given the coordinates of the end of the vector and want to find its magnitude, v , and angle, θ . Because of your knowledge of trigonometry, you know

$$\frac{y}{x} = \frac{v \sin \theta}{v \cos \theta} = \tan \theta$$

Where $\tan \theta$ is the tangent of the angle. This means that

$$\theta = \tan^{-1}(y/x)$$

This formula always gives you an angle between -90° and 90° . If your vector is pointing between 90° and 270° , you'll need to add 180° to the result.

Suppose that the coordinates of the vector are (3, 4). You can find the angle θ as the $\tan^{-1}(4/3) = 53^\circ$.

You can use the Pythagorean theorem to find the *hypotenuse* — the magnitude, v — of the triangle formed by x , y , and v :

$$v = \sqrt{x^2 + y^2}$$

Plug in the numbers for this example to get

$$v = \sqrt{3^2 + 4^2} = 5$$

So if you have a vector given by the coordinates (3, 4), its magnitude is 5, and its angle is 53° .



EXAMPLE

Q. Convert the vector given by the coordinates (1.0, 5.0) into magnitude/angle format.

A. The correct answer is magnitude 5.1, angle 79° .

1. Apply the Pythagorean theorem to find the magnitude. Plug in the numbers to get 5.1.

2. Apply the equation $\theta = \tan^{-1}(y/x)$ to find the angle. Plug in the numbers to get $\tan^{-1}(5.0/1.0) = 79^\circ$.

9

Convert the vector (5.0, 7.0) into magnitude/angle form.

10

Convert the vector (13.0, 13.0) into magnitude/angle form.

11 Convert the vector $(-1.0, 1.0)$ into magnitude/angle form.

12 Convert the vector $(-5.0, -7.0)$ into magnitude/angle form.

Adding Vectors Together

You're frequently asked to add vectors when solving physics problems. To add two vectors, you place them head to tail and then find the length and magnitude of the result. The order in which you add the two vectors doesn't matter. For example, suppose you're headed to the big physics convention and have been told that you go 20 miles due north and then 20 miles due east to get there. At what angle is the convention center from your present location, and how far away is it?

You can write these two vectors like this (where east is along the positive x axis):

$$\begin{aligned}(0, 20 \text{ mi}) \\ (20 \text{ mi}, 0)\end{aligned}$$

In this case, you need to add these two vectors, and you can do that just by adding their x and y components separately:

$$\begin{array}{r}(0, 20 \text{ mi}) \\ +(20 \text{ mi}, 0) \\ \hline (20 \text{ mi}, 20 \text{ mi})\end{array}$$

Do the math, and your resultant vector is $(20, 20)$. You've just completed a vector addition. But the question asks for the vector in magnitude/angle terms, not coordinate terms. So what is the magnitude of the vector from you to the physics convention? You can see the situation in Figure 3-6, where you have x and y and want to find v .

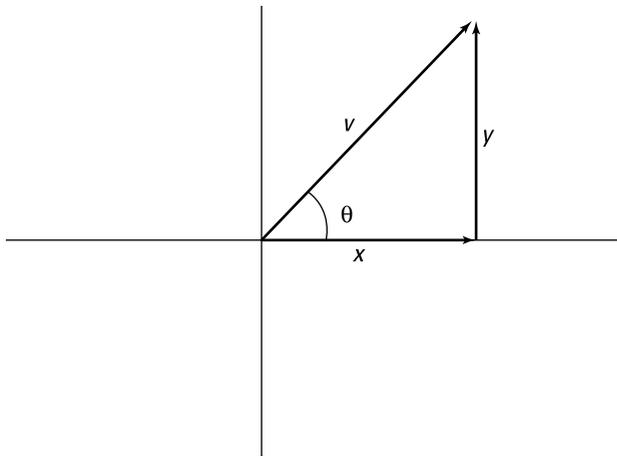


FIGURE 3-6:
Adding two
vectors.

© John Wiley & Sons, Inc.

Finding v isn't so hard because you can use the Pythagorean theorem:

$$v = \sqrt{x^2 + y^2}$$

Plug in the numbers to get

$$v = \sqrt{(20 \text{ mi})^2 + (20 \text{ mi})^2} = 28.3 \text{ miles}$$

So the convention is 28.3 miles away. What about the angle θ ? You know

$$\theta = \tan^{-1}(y/x) = \tan^{-1}(20/20) = 45^\circ$$

And that's it — you now know that the convention is 28.3 miles away at an angle of 45° .



Q. Add the two vectors in Figure 3-7. One has a magnitude 5.0 and angle 45° , and the other has a magnitude 7.0 and angle 35° .

EXAMPLE

A. The correct answer is magnitude 12.0, angle 39° .

1. Resolve the two vectors into their components. For the first vector, apply the equation $v_x = v \cos \theta$ to find the x coordinate. That's $5.0 \cos 45^\circ = 3.5$.
2. Apply the equation $v_y = v \sin \theta$ to find the y coordinate of the first vector. That's $5.0 \sin 45^\circ$, or 3.5. So the first vector is (3.5, 3.5) in coordinate form.
3. For the second vector, apply the equation $v_x = v \cos \theta$ to find the x coordinate. That's $7.0 \cos 35^\circ = 5.7$.
4. Apply the equation $v_y = v \sin \theta$ to find the y coordinate of the second vector. That's $7.0 \sin 35^\circ = 4.0$. So the second vector is (5.7, 4.0) in coordinate form.

5. To add the two vectors, add them in coordinate form: $(3.5, 3.5) + (5.7, 4.0) = (9.2, 7.5)$.
6. Convert $(9.2, 7.5)$ into magnitude/angle form. Apply the equation $\theta = \tan^{-1}(y/x)$ to find the angle, which is $\tan^{-1}(7.5/9.2) = \tan^{-1}(0.82) = 39^\circ$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is $v = \sqrt{9.2^2 + 7.5^2} = 11.9$. Converting to two significant digits gives you 12.

13 Add a vector whose magnitude is 13.0 and angle is 27° to one whose magnitude is 11.0 and angle is 45° .

14 Add a vector whose magnitude is 16.0 and angle is 56° to one whose magnitude is 10.0 and angle is 25° .

15 Add two vectors: Vector one has a magnitude 22.0 and angle of 19° , and vector two has a magnitude 19.0 and an angle of 48° .

16 Add a vector whose magnitude is 10.0 and angle is 257° to one whose magnitude is 11.0 and angle is 105° .

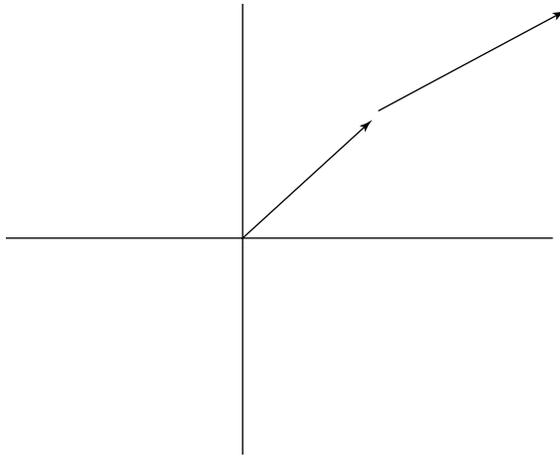


FIGURE 3-7:
Two vectors
being added.

© John Wiley & Sons, Inc.

Handling Velocity as a Vector

Suppose you're in a car traveling east at 88 meters/second when you begin to accelerate north at 5.0 meters/second² for 10.0 seconds. What is your final speed?

You may think you can use this equation to figure out the answer:

$$v_f = v_o + a \cdot t$$

But that's not a vector equation; the quantities here are called *scalars* (the magnitude of a vector is a scalar). This is a scalar equation, and it's not appropriate to use here because the acceleration and the initial speed aren't in the same direction. In fact, speed itself is a scalar, so you have to think in terms not of speed but of velocity.

Velocity is a vector, and as such, it has a magnitude and a direction associated with it. Here's the same equation as a vector equation:

$$\mathbf{v}_f = \mathbf{v}_o + \mathbf{a} \cdot t$$

Note that the speeds are now velocities (speed is the magnitude of a velocity vector) and that everything here is a vector except time (which is always a scalar). This change means that the addition you perform in this equation is vector addition, which is what you want because vectors can handle addition in multiple dimensions, not just in a straight line.



REMEMBER

Here are the equations of motion, written as vector equations:

$$\mathbf{v}_f = \mathbf{v}_o + \mathbf{a} \cdot t$$

$$\mathbf{v} = \frac{\Delta \mathbf{x}}{\Delta t} = \frac{\mathbf{x}_f - \mathbf{x}_o}{t_f - t_o}$$

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\mathbf{v}_f - \mathbf{v}_o}{t_f - t_o}$$

$$\mathbf{s} = \mathbf{v}_o(t_f - t_o) + \frac{1}{2}\mathbf{a}(t_f - t_o)^2$$



EXAMPLE

Q. You're in a car traveling east at 88.0 meters/second; then you accelerate north at 5.00 meters/second² for 10.0 seconds. What is your final speed?

A. The correct answer is 101 meters/second.

1. Start with this vector equation:

$$\mathbf{v}_f = \mathbf{v}_o + \mathbf{a} \cdot t$$

2. This equation is simply vector addition, so treat the quantities involved as vectors. That is, $\mathbf{v}_o = (88, 0)$ meters/second and $\mathbf{a} = (0, 5)$ meters/second². Here's what the equation looks like when you plug in the numbers:

$$\mathbf{v}_f = (88.0, 0) \text{ m/s} + (0, 5.00)(10.0) \text{ m/s}$$

3. Do the math:

$$\mathbf{v}_f = (88.0, 0) \text{ m/s} + (0, 5.00)(10.0) \text{ m/s} = (88.0, 50.0) \text{ m/s}$$

4. You're asked to find the final speed, which is the magnitude of the velocity. Plug your numbers into the Pythagorean theorem.

5. You can also find the final direction. Apply the equation $\theta = \tan^{-1}(y/x)$ to find the angle, which is $\tan^{-1}(50.0/88.0) = \tan^{-1}(0.57) = 29.6^\circ$ in this case.

17 You're going 40.0 meters/second east, and then you accelerate 10.0 meters/second² north for 10.0 seconds. What are the direction and magnitude of your final velocity?

18 You're going 44.0 meters/second at 35°, and then you accelerate due west at 4.0 meters/second² for 20.0 seconds. What are the direction and magnitude of your final velocity?

19 A hockey puck is going 100.0 meters/second at 250° when it's hit by a hockey stick, which accelerates it at 1.0×10^3 meters/second² at 19° for 0.10 second. What are the direction and magnitude of the puck's final velocity?

20 A car is driving along an icy road at 10.0 meters/second at 0° when it skids, accelerating at 15 meters/second² at 63° for 1.0 second. What are the direction and magnitude of the car's final velocity?

Answers to Problems about Vectors

The following are the answers to the practice questions presented in this chapter. You see how to work out each answer, step by step.

1 45 meters at 0°

The marble ends up 45 meters to the right of the origin, which is to say it ends up 45 meters from the origin, at 0° .

2 45 meters at 180°

1. The marble moved 45 meters to the right at 0° and then moved left for 90 meters.
2. The marble ends up at 45 meters on the other side of the origin, at 45 meters at 180° .

3 (5.0, 0) cm

The marble moves to the right — in the positive x direction — by 5.0 meters, so its final location is (5.0, 0).

4 (3.5, 5.6) m

1. You move to the right of the origin by 3.5 meters, leaving you at (3.5, 0) m.
2. Then you move up (in the positive y direction) by 5.6 meters, leaving you at (3.5, 5.6) m.

5 (2.9, 0.8) m

1. Apply the equation $v_x = v \cos \theta$ to find the x coordinate: $3.0 \text{ m} \cdot \cos 15^\circ$, or 2.9 m.
2. Apply the equation $v_y = v \sin \theta$ to find the y coordinate: $3.0 \text{ m} \cdot \sin 15^\circ$, or 0.8 m.

6 (7.4, 5.2) m

1. Apply the equation $v_x = v \cos \theta$ to find the x coordinate: $9.0 \text{ m} \cdot \cos 35^\circ$, or 7.4 m.
2. Apply the equation $v_y = v \sin \theta$ to find the y coordinate: $9.0 \text{ m} \cdot \sin 35^\circ$, or 5.2 m.

7 (-3.4, 4.9) m

1. Apply the equation $v_x = v \cos \theta$ to find the x coordinate: $6.0 \text{ m} \cdot \cos 125^\circ$, or -3.4 m.
2. Apply the equation $v_y = v \sin \theta$ to find the y coordinate: $6.0 \text{ m} \cdot \sin 125^\circ$, or 4.9 m.

8 (-1.0, -3.9) m

1. Apply the equation $v_x = v \cos \theta$ to find the x coordinate: $4.0 \text{ m} \cdot \cos 255^\circ$, or -1.0 m.
2. Apply the equation $v_y = v \sin \theta$ to find the y coordinate: $4.0 \text{ m} \cdot \sin 255^\circ$, or -3.9 m.

9 Magnitude 8.6, angle 54°

1. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is 8.6.
2. Apply the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(7.0 / 5.0) = 54^\circ$.

10 Magnitude 18.4, angle 45°

1. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is 18.4.
2. Apply the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(13.0 / 13.0) = 45^\circ$.

11 Magnitude 1.4, angle 135°

1. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is 1.4.
2. Apply the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(1.0 / -1.0) = -45^\circ$. However, note that the angle must really be between 90° and 180° because the first vector component is negative and the second is positive. That means you should add 180° to -45° , giving you 135° (the tangent of 135° is also $1.0 / -1.0 = -1.0$).

12 Magnitude 8.6, angle 234°

1. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is 8.6.
2. Apply the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(-7.0 / -5.0) = 54^\circ$. However, note that the angle must really be between 180° and 270° because both vector components are negative. That means you should add 180° to 54° , giving you 234° (the tangent of 234° is also $-7.0 / -5.0 = 7.0 / 5.0$).

13 Magnitude 23.7, angle 35°

1. For the first vector, use the equation $v_x = v \cos \theta$ to find the x coordinate:
 $13.0 \cdot \cos 27^\circ = 11.6$.
2. Use the equation $v_y = v \sin \theta$ to find the y coordinate of the first vector: $13.0 \cdot \sin 27^\circ$, or 5.90. So the first vector is (11.6, 5.90) in coordinate form.
3. For the second vector, use the equation $v_x = v \cos \theta$ to find the x coordinate:
 $11.0 \cdot \cos 45^\circ = 7.78$.
4. Use the equation $v_y = v \sin \theta$ to find the y coordinate of the second vector:
 $11.0 \cdot \sin 45^\circ = 7.78$. So the second vector is (7.78, 7.78) in coordinate form.
5. Add the two vectors in coordinate form: $(11.6, 5.90) + (7.78, 7.78) = (19.4, 13.7)$.
6. Convert (19.4, 13.7) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(13.7 / 19.4) = \tan^{-1}(0.71) = 35^\circ$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is $v = \sqrt{19.4^2 + 13.7^2} = 23.7$.

14 Magnitude 25.1, angle 44°

1. For the first vector, use the equation $v_x = v \cos \theta$ to find the x coordinate:
 $16.0 \cdot \cos 56^\circ = 8.95$.
2. Use the equation $v_y = v \sin \theta$ to find the y coordinate of the first vector: $16.0 \cdot \sin 56^\circ$, or 13.3. So the first vector is (8.95, 13.3) in coordinate form.
3. For the second vector, use the equation $v_x = v \cos \theta$ to find the x coordinate:
 $10.0 \cdot \cos 25^\circ = 9.06$.
4. Use the equation $v_y = v \sin \theta$ to find the y coordinate of the second vector:
 $10.0 \cdot \sin 25^\circ = 4.23$. So the second vector is (9.06, 4.23) in coordinate form.
5. Add the two vectors in coordinate form: $(8.95, 13.3) + (9.06, 4.23) = (18.0, 17.5)$.
6. Convert the vector (18.0, 17.5) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(17.5 / 18.0) = \tan^{-1}(0.97) = 44^\circ$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is $\sqrt{18.0^2 + 17.5^2} = 25.1$.

15 Magnitude 39.7, angle 32°

1. For the first vector, use the equation $v_x = v \cos \theta$ to find the x coordinate:
 $22.0 \cdot \cos 19^\circ = 20.8$.
2. Use the equation $v_y = v \sin \theta$ to find the y coordinate of the first vector: $22.0 \cdot \sin 19^\circ$, or 7.16. So the first vector is (20.8, 7.16) in coordinate form.
3. For the second vector, use the equation $v_x = v \cos \theta$ to find the x coordinate:
 $19.0 \cdot \cos 48^\circ = 12.7$.
4. Use the equation $v_y = v \sin \theta$ to find the y coordinate of the second vector:
 $19.0 \cdot \sin 48^\circ = 14.1$. So the second vector is (12.7, 14.1) in coordinate form.
5. Add the two vectors in coordinate form: $(20.8, 7.16) + (12.7, 14.1) = (33.5, 21.3)$.
6. Convert the vector (33.5, 21.3) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(21.3/33.5) = \tan^{-1}(0.64) = 32^\circ$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is $v = \sqrt{33.5^2 + 21.3^2} = 39.7$.

16 Magnitude 5.2, angle 170°

1. For the first vector, use the equation $v_x = v \cos \theta$ to find the x coordinate:
 $10.0 \cdot \cos 257^\circ = -2.25$.
2. Use the equation $v_y = v \sin \theta$ to find the y coordinate of the first vector: $10.0 \cdot \sin 257^\circ$, or -9.74. So the first vector is (-2.25, -9.74) in coordinate form.
3. For the second vector, use the equation $v_x = v \cos \theta$ to find the x coordinate:
 $11.0 \cdot \cos 105^\circ = -2.85$.
4. Use the equation $v_y = v \sin \theta$ to find the y coordinate of the second vector:
 $11.0 \cdot \sin 105^\circ = 10.6$. So the second vector is (-2.85, 10.6) in coordinate form.
5. Add the two vectors in coordinate form: $(-2.25, -9.74) + (-2.85, 10.6) = (-5.10, 0.86)$.
6. Convert the vector (-5.10, 0.86) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(0.86/-5.10) = \tan^{-1}(-0.17) = 170^\circ$. Because x is negative and y is positive, this vector must be in the second quadrant.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude, which is $v = \sqrt{5.10^2 + 0.86^2} = 5.2$.

17 Magnitude 108 meters/second, angle 68°

1. Start with this equation: $\mathbf{v}_f = \mathbf{v}_o + \mathbf{a} \cdot t$.
2. Plug in the numbers: $\mathbf{v}_f = (40.0, 0) \text{ m/s} + (0, 10.0)(10.0) \text{ m/s} = (40.0, 100.0) \text{ m/s}$.
3. Convert the vector (40.0, 100.0) into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(100.0/40.0) = \tan^{-1}(2.5) = 68^\circ$.
4. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the speed — the magnitude of the velocity, giving you 108 meters/second.

18 Magnitude 50.7 meters/second, angle 150°

1. Start with this equation: $\mathbf{v}_f = \mathbf{v}_o + \mathbf{a} \cdot t$.
2. Convert the original velocity into vector component notation. Use the equation $v_x = v \cos \theta$ to find the x coordinate of the original velocity vector: $44.0 \text{ m/s} \cdot \cos 35^\circ = 36.0 \text{ m/s}$.

- Use the equation $v_y = v \sin \theta$ to find the y coordinate of the velocity: $44.0 \text{ m/s} \cdot \sin 35^\circ$, or 25.2 m/s . So the velocity is $(36.0, 25.2) \text{ m/s}$ in coordinate form.
- Perform the vector addition: $(36.0, 25.2) \text{ m/s} + (-4.0, 0)(20.0) \text{ m/s} = (-44.0, 25.2) \text{ m/s}$.
- Convert the vector $(-44.0, 25.2) \text{ m/s}$ into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(25.2 / -44.0) = \tan^{-1}(0.57) = 150^\circ$.
- Apply the equation $v = \sqrt{x^2 + y^2}$ to find the speed — the magnitude of the velocity, giving you $50.7 \text{ meters/second}$.

19 Magnitude 86.1 meters/second, angle -46°

- Start with this equation: $\mathbf{v}_f = \mathbf{v}_o + \mathbf{a} \cdot t$.
- Convert the original velocity into vector component notation. Use the equation $v_x = v \cos \theta$ to find the x coordinate of the original velocity vector: $100.0 \text{ m/s} \cdot \cos 250^\circ = -34.2 \text{ m/s}$.
- Use the equation $v_y = v \sin \theta$ to find the y coordinate of the velocity: $100.0 \text{ m/s} \cdot \sin 250^\circ$, or -94.0 m/s . So the original velocity is $(-34.2, -94.0) \text{ m/s}$ in coordinate form.
- Convert the acceleration into components. Use the equation $a_x = a \cos \theta$ to find the x coordinate of the acceleration: $(1.0 \times 10^3) \text{ m/s}^2 \cos 19^\circ = 946 \text{ m/s}^2$.
- Use the equation $a_y = a \sin \theta$ to find the y coordinate of the acceleration: $(1.0 \times 10^3 \text{ m/s}^2) \sin 19^\circ$, or 325 m/s^2 . So the acceleration is $(946, 325) \text{ m/s}^2$ in coordinate form.
- Perform the vector addition: $(-34.2, -94.0) \text{ m/s} + (946, 325)(0.1) \text{ m/s} = (60.3, -61.5) \text{ m/s}$.
- Convert the vector $(60.3, -61.5) \text{ m/s}$ into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(61.5 / 60.3) = \tan^{-1}(-1.0) = -46^\circ$.
- Apply the equation $v = \sqrt{x^2 + y^2}$ to find the speed — the magnitude of the velocity, giving you $86.1 \text{ meters/second}$.

20 Magnitude 21.5 meters/second, angle 39°

- Start with this equation: $\mathbf{v}_f = \mathbf{v}_o + \mathbf{a} \cdot t$.
- Convert the original velocity into vector component notation: $(10.0, 0) \text{ meters/second}$.
- Convert the acceleration into components. Use the equation $a_x = a \cos \theta$ to find the x coordinate of the acceleration: $15 \text{ m/s}^2 \cdot \cos 63^\circ = 6.8 \text{ m/s}^2$.
- Use the equation $a_y = a \sin \theta$ to find the y coordinate of the acceleration: $15 \text{ m/s}^2 \cdot \sin 63^\circ$, or 13.4 m/s^2 . So the acceleration is $(6.8, 13.4) \text{ m/s}^2$ in coordinate form.
- Perform the vector addition: $(10.0, 0) \text{ m/s} + (6.8, 13.4)(1.0) \text{ m/s} = (16.8, 13.4) \text{ m/s}$.
- Convert the vector $(16.8, 13.4)$ into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(13.4 / 16.8) = \tan^{-1}(0.79) = 39^\circ$.
- Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the velocity, giving you $21.5 \text{ meters/second}$.



**May the Forces
Be with You**

IN THIS PART . . .

Nail down Newton's laws of motion to understand how every action has an equal and opposite reaction.

See how gravitational force and friction affect objects moving down an inclined plane.

Wrap your mind around what happens when objects move in a circle.

IN THIS CHAPTER

- » Applying Newton's laws of motion
- » Calculating force and acceleration
- » Working with weight and mass
- » Keeping things in balance with equilibrium

Chapter 4

Applying Force

If all you have is objects in motion, things get pretty boring — everything zipping around endlessly in straight lines, never changing direction or speed. So physics comes to the rescue by applying forces to those objects, and the application of force means changes in the objects' speed and direction. Now that's much more interesting.

This chapter covers Isaac Newton's three laws of motion. More specifically, it examines force and the application of it on an object.

Newton's First Law of Motion



REMEMBER

Whether you're asked to give it in physics class or not, you need to be familiar with Isaac Newton's First Law of Motion: "An object continues in a state of rest or in a state of motion at a constant speed along a straight line, unless compelled to change that state by a net force." What's the translation? The idea is that if you don't apply a force to something in motion, it will stay in that same motion along a straight line. Forever!

And what is force? Simply defined, *force* is a push or a pull on an object. When you pull on a wagon, you're applying a force to that wagon. When you push a shopping cart in the grocery store, you're applying a force to that shopping cart.

The property of an object to stay in constant motion is called *inertia*, and mass is a measure of that inertia. In physics, it's very important to realize that there's a difference between mass and weight; objects have mass built in, but they don't acquire weight until you put them in a gravitational field.

Physics measures mass using the unit kilograms in the MKS system and grams in the CGS system. What's the unit of mass in the FPS (foot-pound-second) system? It's the oddly named *slug*. On Earth's surface, a slug weighs approximately 32 pounds (you may not have heard of FPS slugs, but you've surely heard of pounds). (For a review of measurement systems, flip to Chapter 1.)



EXAMPLE

Q. Suppose that you have an SUV and want to measure its mass in the MKS system. What unit of measurement do you use?

A. The correct answer is kilograms because the MKS unit of mass is the kilogram.

1 You decide to measure the mass of an aircraft carrier in the CGS system. What unit does your measurement end up in?

2 Your sister asks you if she looks fat, so you decide to measure her mass in the FPS system. What unit is your measurement in?

Newton's Second Law of Motion

Force saves you from the monotony of everything moving at the same speed and direction forever. Force can act on objects, changing their direction and/or speed. The relationship between force, mass, and acceleration is primary in physics classes. This section (and even broader, this chapter) helps you become a pro at solving problems involving these quantities.



REMEMBER

To start, you need to know Newton's Second Law of Motion, which is a big one in physics: "When a net force ΣF acts on an object of mass m , the acceleration of that mass can be calculated by $\Sigma F = ma$." The Σ stands for "sum," so $\Sigma F = ma$ can be read as "the sum of all forces on an object (the net force) equals mass times acceleration." This equation is often just abbreviated as $F = ma$. It's important to note that ΣF refers to the sum of all the forces on the object and not to just a single force.

What is the unit of force? Table 4-1 gives you a rundown for the three measurement systems:

Table 4-1 **Units of Force**

System	Unit	Name	Abbreviation
MKS	kg-m/s ²	newton	N
CGS	g-m/s ²	dyne	dyn
FPS	pound		lb

Here's how you relate the three different units of force:

$$1 \text{ lb} = 4.48 \text{ N}$$

$$1 \text{ N} = 10^5 \text{ dyn}$$



EXAMPLE

Q. You're at rest on an ice rink when you get hit from behind with a force of 50 N as someone bumps you. If your mass is 70 kg, what is your acceleration?

A. The correct answer is 0.714 m/s².

1. Use the equation $F = ma$: Solving for a gives you $a = F/m$.

2. Plug in the numbers: $a = F/m = (50.0 \text{ N})/(70.0 \text{ kg}) = 0.714 \text{ m/s}^2$.

3 You come home to find a delivered package with a mass of 100 kg blocking the door. If you push it with a force of 100 N, what will its acceleration be if no friction is involved?

4 You're gliding across a frictionless lake in a sailboat. If your mass is 70 kg and the boat's mass is 200 kg, with what force does the wind need to blow you to give you an acceleration of 0.30 m/s²?

5 You have control of a space station, which has a mass of 400,000 kg. To give it an acceleration of 2.0 m/s^2 , what force do you need to apply with the rockets?

6 You find a stone in the forest and give it a push of 50.0 N. It accelerates at 2.0 m/s^2 . What is its mass?

7 You're applying a force of 17 N to a hockey puck with a mass of 0.17 kg. Starting from rest, how far has the puck gone in 0.10 seconds?

8 You push a rowboat on a calm lake (assuming no friction) with a force of 40.0 N. If the rowboat has a mass of 80.0 kg, how far has it gone in 10.0 seconds?

9 A space station with a mass of 1.0×10^5 kg is moving toward a satellite at 5.0 m/s. If you want to avoid crashing them together and have only 1.0×10^3 seconds in which to act, what force do you need to apply to stop the space station from colliding with the satellite?

10 Your 1,000-kg car needs a push. Starting at rest, how hard do you have to push to get it up to 5.0 m/s in 1.0×10^2 seconds?

Force Is a Vector

Force, like displacement, velocity, and acceleration, is a vector quantity, which is why Newton's Second Law is written as $\Sigma F = ma$. Put into words, it says that the vector sum of the forces acting on an object is equal to its mass (a scalar) multiplied by its acceleration (a vector).

Because force is a vector quantity, you add forces together as vectors. That fits right into Newton's Second Law. (For more on the process of adding vectors, check out Chapter 3.)



Q. Suppose that you have two forces as shown in Figure 4-1: **A** = 5.0 N at 40° , and **B** = 7.0 N at 125° . What is the net force, ΣF ?

EXAMPLE

A. The correct answer is magnitude 8.9 N, angle 91° .

1. Convert force **A** into vector component notation. Use the equation $A_x = A \cos \theta$ to find the x coordinate of the force: $5.0 \text{ N} \cos 40^\circ = 3.8 \text{ N}$.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of the force: $5.0 \text{ N} \sin 40^\circ$, or 3.2 N. That makes the vector **A** (3.8, 3.2) N in coordinate form.

3. Convert the vector **B** into components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of the acceleration: $7.0 \text{ N} \cos 125^\circ = -4.0 \text{ N}$.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of the second force: $7.0 \text{ N} \sin 125^\circ$, or 5.7 N . That makes the force **B** $(-4.0, 5.7) \text{ N}$ in coordinate form.
5. Perform the vector addition to find the net force:
 $(3.8, 3.2) \text{ N} + (-4.0, 5.7) \text{ N} = (-0.2, 8.9) \text{ N}$
6. Convert the vector $(-0.2, 8.9)$ into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(-44.5) = 91^\circ$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 8.9 N .

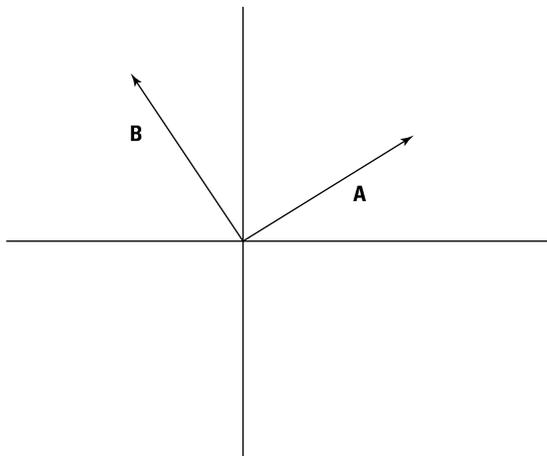


FIGURE 4-1:
Two forces.

© John Wiley & Sons, Inc.

- 11 Add two forces: **A** is 8.0 N at 53° , and **B** is 9.0 N at 19° .

- 12 Add two forces: **A** is 16.0 N at 39° , and **B** is 5.0 N at 125° .

- 13 Add two forces: **A** is 22.0 N at 68°, and **B** is 6.0 N at 24°.

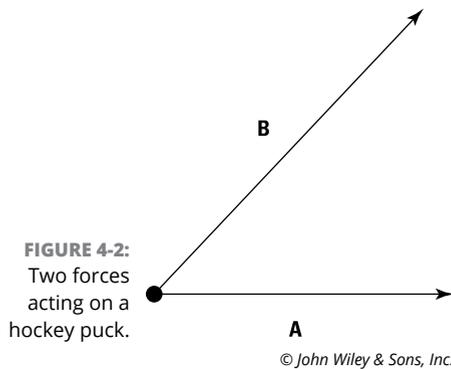
- 14 Add two forces: **A** is 12.0 N at 129°, and **B** is 3.0 N at 225°.

Calculating Net Force and Acceleration

Newton says $\Sigma F = ma$, which means that you add all the force vectors together to get the net force.

That's how it usually works when you have to figure out $F = ma$ problems in physics. Most of the time, a number of force vectors are involved, and you have to solve for the net force to find the acceleration.

Take a look at the hockey puck in Figure 4-2. Two forces, **A** and **B**, are acting on the puck. What's going to happen to the puck?



You don't have to calculate the result of each force acting separately on the hockey puck because the net force is what's important. Calculate the net force first and then use that in $\Sigma F = ma$.



EXAMPLE

Q. Suppose that the forces acting on the hockey puck in Figure 4-2 are $\mathbf{A} = 9.0 \text{ N}$ at 0° , and $\mathbf{B} = 14.0 \text{ N}$ at 45° . What is the acceleration of the puck, given that its mass is $1.00 \times 10^{-1} \text{ kg}$?

A. The correct answer is magnitude 213 m/s^2 , angle 28° .

1. Convert force \mathbf{A} into vector component notation. Use the equation $A_x = A \cos \theta$ to find the x coordinate of the force: $9.0 \text{ N} \cos 0^\circ = 9.0 \text{ N}$.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of the force: $9.0 \text{ N} \sin 0^\circ$, or 0.0 N . That makes the vector \mathbf{A} $(9.0, 0.0) \text{ N}$ in coordinate form.
3. Convert the vector \mathbf{B} into components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of the acceleration: $14.0 \text{ N} \cos 45^\circ = 9.9 \text{ N}$.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of the second force: $14.0 \text{ N} \sin 45^\circ$, or 9.9 N . That makes the force \mathbf{B} $(9.9, 9.9) \text{ N}$ in coordinate form.
5. Perform the vector addition to find the net force:
 $(9.0, 0.0) \text{ N} + (9.9, 9.9) \text{ N} = (18.9, 9.9) \text{ N}$.
6. Convert the vector $(18.9, 9.9) \text{ N}$ into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle of the net force: $\tan^{-1}(0.52) = 28^\circ$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 21.3 N .
8. Convert 21.3 N into acceleration: $a = F/m = (21.3 \text{ N}) / (0.100 \text{ kg}) = 213 \text{ m/s}^2$.

15 Assume that the two forces acting on a 0.10-kg hockey puck are as follows: \mathbf{A} is 16.0 N at 53° , and \mathbf{B} is 21.0 N at 19° . What is the acceleration of the hockey puck?

16 Two forces act on a $1.0 \times 10^3 \text{ kg}$ car. \mathbf{A} is 220 N at 64° , and \mathbf{B} is 90.0 N at 80° . What is the car's acceleration?

17 Suppose that two forces act on a 100-kg boat. **A** is 100 N at 10° , and **B** is 190 N at 210° . What is the boat's acceleration?

18 A marble with a mass of 1.0 g is hit by two other marbles that each apply a force for 0.3 seconds. If force **A** is 0.010 N at 63° and **B** is 0.050 N at 135° , what is the acceleration of the original marble?

Sorting Out Weight and Mass

One of the forces you're asked to deal with frequently in physics is the force on an object due to gravity. Near Earth or another large mass (such as the moon), all objects are accelerated due to gravity; on the surface of the Earth, the acceleration due to gravity is 9.8 m/s^2 down, which is about 32 ft/s^2 down.

Because the equation $\Sigma F = ma$ holds for all forces, you can calculate the force an object feels due to gravity. Here are a few things to keep in mind when dealing with force and gravity:

- » **The force on an object is proportional to the object's mass.** For example, twice the mass means twice the force.
- » **Because $a = F/m$, twice the force still means the same acceleration if you have twice the mass.** The acceleration due to gravity is constant near the Earth's surface, not the force.
- » **The acceleration due to gravity points downward, toward the center of the Earth.**

The acceleration due to gravity at the Earth's surface carries the symbol g . That means $\Sigma F = ma$ becomes $F = mg$ when the only force on an object is gravity; as a vector equation, that's $F = mg$, where the vector g is pointing downward. In practical terms, unless you're dealing with points so far apart that the curvature of the Earth matters, g is just considered as always acting in the same downward direction.



EXAMPLE

Q. You're trying to lift a suitcase with a mass of 20 kg. How much force must you apply at a minimum to lift the suitcase?

A. The correct answer is 200 N.

To be able to lift the suitcase, you have to overcome the force due to gravity. Calculate it as follows:

$$F = mg = (20 \text{ kg})(9.8 \text{ m/s}^2) = 196 \text{ N}$$

With significant figures, this rounds to 200 N.

19 You're holding a basketball in your hands. If this basketball has a mass of 0.8 kg, how much force must you provide to hold it up against gravity?

20 A ball drops off a cliff. How fast is it going 1.0 second later (neglecting air resistance)?

21 A ball drops off a cliff. How far has it gone in 2.0 seconds (neglecting air resistance)?

22 A sky diver jumps out of a plane. How far has the sky driver gone 4.0 seconds later (neglecting air resistance)?

23 You throw a baseball straight up into the air at 60 m/s. How fast is it going 1.0 second later (neglecting forces from the air)?

24 You throw a physics book straight up into the air at 30 m/s. How fast is it going 4.0 seconds later (neglecting air resistance)?

25 Holding a ball over the edge of a cliff, you throw it up at 10 m/s. What is its position 5.0 seconds later (neglecting forces from the air)?

26 Holding a ball over the edge of a cliff, you throw it up at 20 m/s. What is its position 7.0 seconds later (neglecting forces from the air)?

The Balancing Act of Equilibrium

When you place a book on a table, it doesn't accelerate down toward Earth, even though Earth's gravity is pulling the book down, because a second force, from the table, is pushing up on the book and balancing the force from gravity. The book is in equilibrium. An object is in equilibrium if all the forces on it exactly cancel out.



EXAMPLE

Q. Figure 4-3 shows three connected ropes. If rope A has a tension of 10 N at 135° and rope B has a tension of 10 N at 45° , what must the tension in rope C be to keep things in equilibrium?

A. The correct answer is 14 N downward.

1. Convert tension **A** into vector component notation. Use the equation $A_x = A \cos \theta$ to find the x coordinate of the tension from rope A: $10.0 \text{ N} \cos 135^\circ = -7.07 \text{ N}$.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of the tension from rope A: $10.0 \text{ N} \sin 135^\circ$, or 7.07 N. That makes the tension **A** $(-7.07, 7.07)$ N in coordinate form.
3. Convert the tension **B** into components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of the tension from rope B: $10.0 \text{ N} \cos 45^\circ = 7.07 \text{ N}$.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of the second tension: $10.0 \text{ N} \sin 45^\circ$, or 7.07 N. That makes the tension **B** $(7.07, 7.07)$ N in coordinate form.
5. Perform vector addition to find the net tension:
 $(-7.07, 7.07) \text{ N} + (7.07, 7.07) \text{ N} = (0, 14.1) \text{ N}$.
6. To counteract the total tension from ropes A and B, the tension in rope C must be 14.1 N downward (that is, -14.1 N). With significant figures, this answer rounds to 14 N downward.

27 You have three ropes tied together in equilibrium. The tension in rope A is 15 N at 135° , and the tension in rope B is 15 N at 45° . What must the tension in rope C be?

28 You have three ropes tied together in equilibrium. The tension in rope A is 17.0 N at 115° , and the tension in rope B is 18.0 N at 25° . What must the tension in rope C be?

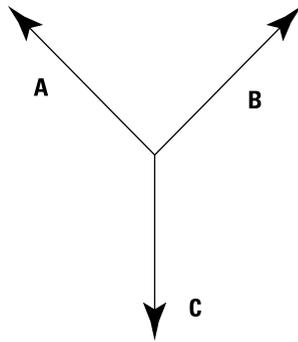


FIGURE 4-3:
Three forces
balancing out.

© John Wiley & Sons, Inc.

Newton's Third Law of Motion

Newton's Third Law of Motion is a famous one: "Whenever one body exerts a force on a second body, the second body exerts an oppositely directed force of equal magnitude on the first body." If that doesn't ring a bell, try this on for size: "For every action, there is an equal and opposite reaction."

Put simply, this law of motion says that if your car pushes against Earth, Earth pushes back against your car with the same amount of force. Similarly, if you hold an apple in the air and release it, the Earth's gravitational force causes the apple to accelerate downward. What about Earth? Doesn't it accelerate upward in response to the force the apple exerts on it? Believe it or not, it does, but because Earth has a mass of about 60,000,000,000,000,000,000,000,000 times that of the apple, the effect of the apple on Earth isn't very noticeable!

Note: The "action force" doesn't come before the "reaction force," and an "action" doesn't cause a "reaction." The two act simultaneously as a single interaction. The terms *action* and *reaction* are arbitrary. When you push on a wall, you could say that the action force is you pushing on the wall and the reaction force is the wall pushing back on you. You can also (just as correctly) state that the action force is the wall pushing on you and the reaction force is you pushing back on the wall.

Answers to Problems about Force

The following are the answers to the practice questions presented in this chapter. You see how to work out each answer, step by step.

1 grams

The units of mass in the CGS system are grams.

2 slugs

The units of mass in the FPS system are slugs.

3 1 m/s^2

1. Solving $F = ma$ for a gives you $a = F/m$.
2. Plug in the numbers: $a = F/m = 100 \text{ N}/100 \text{ kg} = 1 \text{ m/s}^2$.

4 81 N

1. Use the equation $F = ma$.
2. Plug in the numbers: $F = ma = (70 \text{ kg} + 200 \text{ kg})(0.30 \text{ m/s}^2) = 81 \text{ N}$.

5 800,000 N

1. Use the equation $F = ma$.
2. Plug in the numbers: $F = ma = (400,000 \text{ kg})(2.0 \text{ m/s}^2) = 800,000 \text{ N}$.

6 25 kg

1. Use the equation $F = ma$, and solve for the mass, giving you $m = F/a$.
2. Plug in the numbers: $m = F/a = 50 \text{ N}/2.0 \text{ m/s}^2 = 25 \text{ kg}$.

7 0.50 m

1. Use the equation $F = ma$, and solve for the acceleration, giving you $a = F/m$.
2. Use the equation $s = \frac{1}{2}at^2$, and substitute F/m for a :

$$s = \frac{Ft^2}{2m}$$

3. Plug in the numbers: $s = \frac{Ft^2}{2m} = \frac{17 \text{ N} \cdot (0.10 \text{ s})^2}{2(0.17 \text{ kg})} = 0.50 \text{ m}$

8 25 m

1. Use the equation $F = ma$, and solve for the acceleration, giving you $a = F/m$.
2. Use the equation $s = \frac{1}{2}at^2$ and substitute F/m for a :

$$s = \frac{Ft^2}{2m}$$

3. Plug in the numbers: $s = \frac{Ft^2}{2m} = \frac{40.0 \text{ N} \cdot (10.0 \text{ s})^2}{2(80.0 \text{ kg})} = 25 \text{ m}$

9 $-5.0 \times 10^2 \text{ N}$

1. Use the equation $F = ma$, and solve for the acceleration, giving you $a = F/m$.
2. Use the equation $v_f = v_o + at$. In this question, $v_f = 0$, so $at = -v_o$.

- This becomes $at = \frac{Ft}{m} = -v_o$, solving for F gives you $F = \frac{-mv_o}{t}$.
- Plug in the numbers, and you get $F = -500 \text{ N} = -5.0 \times 10^2 \text{ N}$ (it's negative because it's opposite to the direction of travel).

10 **50 N**

- Use the equation $F = ma$, and solve for the acceleration, giving you $a = F/m$.
- Use the equation $v_f = v_o + at$. In this question, $v_f = 10$ and $v_o = 0$, so $at = v_f$.
- This becomes $at = \frac{Ft}{m} = v_f$; solving for F gives you $F = \frac{mv_f}{t}$.
- Plug in the numbers, and you get $F = 50 \text{ N}$.

11 **Magnitude: 16 N; Angle: 35°**

- Convert force **A** into vector component notation. Use the equation $A_x = A \cos \theta$ to find the x coordinate of force **A**: $8.0 \text{ N} \cos 53^\circ = 4.8 \text{ N}$.
- Use the equation $A_y = A \sin \theta$ to find the y coordinate of force **A**: $8.0 \text{ N} \sin 53^\circ = 6.4 \text{ N}$. That makes force **A** (4.8, 6.4) N in coordinate form.
- Convert the vector **B** into components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of force **B**: $9.0 \text{ N} \cos 19^\circ = 8.5 \text{ N}$.
- Use the equation $B_y = B \sin \theta$ to find the y coordinate of the second force: $9.0 \text{ N} \sin 19^\circ = 2.9 \text{ N}$. That makes force **B** (8.5, 2.9) N in coordinate form.
- Perform vector addition to find the net force: $(4.8, 6.4) \text{ N} + (8.5, 2.9) \text{ N} = (13.3, 9.3) \text{ N}$.
- Convert the force vector (13.3, 9.3) N into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(0.70) = 35^\circ$.
- Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 16 N.

12 **Magnitude: 17 N; Angle: 56°**

- Convert force **A** into vector component notation. Use the equation $A_x = A \cos \theta$ to find the x coordinate of force **A**: $16.0 \text{ N} \cos 39^\circ = 12.4 \text{ N}$.
- Use the equation $A_y = A \sin \theta$ to find the y coordinate of force **A**: $16.0 \text{ N} \sin 39^\circ = 10.0 \text{ N}$. That makes force **A** (12.4, 10.0) N in coordinate form.
- Convert force **B** into components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of force **B**: $5.0 \text{ N} \cos 125^\circ = -2.9 \text{ N}$.
- Use the equation $B_y = B \sin \theta$ to find the y coordinate of the second force: $5.0 \text{ N} \sin 125^\circ = 4.1 \text{ N}$. That makes force **B** (-2.9, 4.1) N in coordinate form.
- Perform vector addition to find the net force: $(12.4, 10.0) \text{ N} + (-2.9, 4.1) \text{ N} = (9.5, 14.1) \text{ N}$.
- Convert the force vector (9.5, 14.1) N into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(1.5) = 56^\circ$.
- Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 17 N.

13 **Magnitude: 27 N; Angle: 59°**

- Convert force **A** into vector component notation. Use the equation $A_x = A \cos \theta$ to find the x coordinate of force **A**: $22.0 \text{ N} \cos 68^\circ = 8.24 \text{ N}$.
- Use the equation $A_y = A \sin \theta$ to find the y coordinate of force **A**: $22.0 \text{ N} \sin 68^\circ = 20.4 \text{ N}$. That makes force **A** (8.24, 20.4) N in coordinate form.

- Convert force **B** into components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of force **B**: $6.0 \text{ N} \cos 24^\circ = 5.5 \text{ N}$.
- Use the equation $B_y = B \sin \theta$ to find the y coordinate of force **B**: $6.0 \text{ N} \sin 24^\circ = 2.4 \text{ N}$. That makes force **B** (5.5, 2.4) N in coordinate form.
- Perform vector addition to find the net force: $(8.24, 20.3) \text{ N} + (5.5, 2.4) \text{ N} = (13.7, 22.7) \text{ N}$.
- Convert the force vector (13.7, 22.7) N into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(1.66) = 59^\circ$.
- Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 27 N.

14 Magnitude: 12 N; Angle: 143°

- Convert force **A** into vector component notation. Use the equation $A_x = A \cos \theta$ to find the x coordinate of force **A**: $12.0 \text{ N} \cos 129^\circ = -7.6$.
- Use the equation $A_y = A \sin \theta$ to find the y coordinate of force **A**: $12.0 \text{ N} \sin 129^\circ = 9.3 \text{ N}$. That makes force **A** (-7.6, 9.3) N in coordinate form.
- Convert force **B** into components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of force **B**: $3.0 \text{ N} \cos 225^\circ = -2.1 \text{ N}$.
- Use the equation $B_y = B \sin \theta$ to find the y coordinate of force **B**: $3.0 \text{ N} \sin 225^\circ = -2.1 \text{ N}$. That makes force **B** (-2.1, -2.1) N in coordinate form.
- Perform vector addition to find the net force: $(-7.6, 9.3) \text{ N} + (-2.1, -2.1) \text{ N} = (-9.7, 7.2) \text{ N}$.
- Convert the force vector (-9.7, 7.2) N into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle: $\tan^{-1}(-0.74) = 143^\circ$.
- Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 12 N.

15 Magnitude: 354 m/s²; Angle: 34°

- Convert force **A** into its components. Use the equation $A_x = A \cos \theta$ to find the x coordinate of force **A**: $16.0 \text{ N} \cos 53^\circ = 9.6 \text{ N}$.
- Use the equation $A_y = A \sin \theta$ to find the y coordinate of force **A**: $16.0 \text{ N} \sin 53^\circ = 12.8 \text{ N}$. That makes force **A** (9.6, 12.8) N in coordinate form.
- Convert force **B** into its components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of force **B**: $21.0 \text{ N} \cos 19^\circ = 19.9 \text{ N}$.
- Use the equation $B_y = B \sin \theta$ to find the y coordinate of force **B**: $21.0 \text{ N} \sin 19^\circ = 6.8 \text{ N}$. That makes force **B** (19.9, 6.8) N in coordinate form.
- Perform vector addition to find the net force: $(9.6, 12.8) \text{ N} + (19.9, 6.8) \text{ N} = (29.5, 19.6) \text{ N}$.
- Convert the vector (29.5, 19.6) N into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle of the net force: $\tan^{-1}(0.66) = 34^\circ$. The direction of the acceleration is the same as the direction of the net force: 34° .
- Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 35.4 N.
- Use the magnitude of the force and the mass to find the magnitude of the acceleration: $a = F/m = (35.4 \text{ N}) / (0.10 \text{ kg}) = 354 \text{ m/s}^2$.

16 Magnitude: 0.31 m/s²; Angle: 69°

1. Convert force **A** into its components. Use the equation $A_x = A \cos \theta$ to find the x coordinate of force **A**: $220 \text{ N} \cos 64^\circ = 96 \text{ N}$.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of the force: $220 \text{ N} \sin 64^\circ = 198 \text{ N}$. That makes force **A** (96, 198) N in coordinate form.
3. Convert force **B** into its components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of force **B**: $90 \text{ N} \cos 80^\circ = 16 \text{ N}$.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of force **B**: $90 \text{ N} \sin 80^\circ = 89 \text{ N}$. That makes force **B** (16, 89) N in coordinate form.
5. Perform vector addition to find the net force: $(96, 198) \text{ N} + (16, 89) \text{ N} = (112, 287) \text{ N}$.
6. Convert the vector (112, 287) N into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle of the net force: $\tan^{-1}(2.56) = 69^\circ$. The direction of the acceleration is the same as the direction of the net force: 69° .
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 308 N.
8. Use the magnitude of the force and the mass to find the magnitude of the acceleration:
 $a = F/m = (308 \text{ N}) / (1,000 \text{ kg}) = 0.31 \text{ m/s}^2$.

17 Magnitude: 1 m/s²; Angle: 229°

1. Convert force **A** into its components. Use the equation $A_x = A \cos \theta$ to find the x coordinate of force **A**: $100 \text{ N} \cos 10^\circ = 98 \text{ N}$.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of force **A**: $100 \text{ N} \sin 10^\circ = 17 \text{ N}$. That makes force **A** (98, 17) N in coordinate form.
3. Convert force **B** into its components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of force **B**: $190 \text{ N} \cos 210^\circ = -165 \text{ N}$.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of force **B**: $190 \text{ N} \sin 210^\circ = -95 \text{ N}$. That makes force **B** (-165, -95) N in coordinate form.
5. Perform vector addition to find the net force: $(98, 17) \text{ N} + (-165, -95) \text{ N} = (-67, -78) \text{ N}$.
6. Convert the vector (-67, -78) N into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle of the net force: $\tan^{-1}(1.2) = 49^\circ$. But that answer's not right because both components are negative, which means that the angle is actually between 180° and 270° . Add 180° to 49° to get 229° . The acceleration is in the same direction as the net force.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you 102 N.
8. Use the magnitude of the force and the mass to find the magnitude of the acceleration:
 $a = F/m = (102 \text{ N}) / (100 \text{ kg}) = 1.0 \text{ m/s}^2$.

18 Magnitude: 54 m/s²; Angle: 125°

1. Convert force **A** into its components. Use the equation $A_x = A \cos \theta$ to find the x coordinate of force **A**: $0.01 \text{ N} \cos 63^\circ = 4.5 \times 10^{-3} \text{ N}$.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of force **A**: $0.01 \text{ N} \sin 63^\circ = 8.9 \times 10^{-3} \text{ N}$. That makes force **A** (4.5×10^{-3} , 8.9×10^{-3}) N in coordinate form.

- Convert force **B** into its components. Use the $B_x = B \cos \theta$ to find the x coordinate of force **B**: $0.05 \text{ N} \cos 135^\circ = -3.5 \times 10^{-2} \text{ N}$.
- Use the equation $B_y = B \sin \theta$ to find the y coordinate of force **B**: $0.05 \text{ N} \sin 135^\circ = 3.5 \times 10^{-2} \text{ N}$. That makes force **B** $(-3.5 \times 10^{-2}, 3.5 \times 10^{-2}) \text{ N}$ in coordinate form.
- Perform vector addition to find the net force: $(4.5 \times 10^{-3}, 8.9 \times 10^{-3}) \text{ N} + (-3.5 \times 10^{-2}, 3.5 \times 10^{-2}) \text{ N} = (-3.1 \times 10^{-2}, 4.4 \times 10^{-2}) \text{ N}$.
- Convert the vector $(-3.1 \times 10^{-2}, 4.4 \times 10^{-2}) \text{ N}$ into magnitude/angle form. Use the equation $\theta = \tan^{-1}(y/x)$ to find the angle of the net force: $\tan^{-1}(1.42) = 125^\circ$. The acceleration is in the same direction as the net force.
- Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of the net force, giving you $5.4 \times 10^{-2} \text{ N}$.
- Use the magnitude of the force and the mass to find the magnitude of the acceleration: $a = F / m = 5.4 \times 10^{-2} \text{ N} / (0.001 \text{ kg}) = 54 \text{ m/s}^2$.

19 8 N

- Use the equation $F = mg$.
- The weight of the basketball is $F = mg = (0.8 \text{ kg})(9.8 \text{ m/s}^2) = 7.8 \text{ N}$. Rounding for significant figures, you get 8 N.

20 9.8 m/s, downward

- The acceleration due to gravity is **g**.
- The velocity of the ball is $\mathbf{v}_t = at = gt$.
- Plug in the numbers: $\mathbf{v}_t = (9.8 \text{ m/s}^2)(1.0 \text{ s}) = 9.8 \text{ m/s}$, downward.

21 20.0 m

- The acceleration due to gravity is **g**.
- The distance the ball has gone in a time t is $s = \frac{1}{2}gt^2$.
- Plug in the numbers: $s = \frac{1}{2}gt^2 = \frac{1}{2}(9.8 \text{ m/s}^2)(2.0 \text{ s})^2 = 20 \text{ m}$.

22 78 m

- The acceleration due to gravity is **g**.
- The distance the sky diver has gone in a time t is $s = \frac{1}{2}gt^2$.
- Plug in the numbers: $s = \frac{1}{2}gt^2 = \frac{1}{2}(9.8 \text{ m/s}^2)(4.0 \text{ s})^2 = 78 \text{ m}$.

23 50 m/s

- The acceleration due to gravity is **g**.
- Use the equation $\mathbf{v}_t = \mathbf{v}_o + at = \mathbf{v}_o + gt$.
- Plug in the numbers: $\mathbf{v}_t = \mathbf{v}_o + at = \mathbf{v}_o + gt = 60 \text{ m/s} - (9.8 \text{ m/s}^2)(1.0 \text{ s}) = 50.2 \text{ m/s}$. With significant figures, the answer rounds to 50 m/s.

24 **-9 m/s**

1. The acceleration due to gravity is g .
2. Use the equation $\mathbf{v}_f = \mathbf{v}_o + at = \mathbf{v}_o + gt$.
3. Plug in the numbers: $\mathbf{v}_f = \mathbf{v}_o + at = \mathbf{v}_o + gt = 30 \text{ m/s} - 9.8 \text{ m/s}^2(4.0 \text{ s}) = -9.2 \text{ m/s}$. With significant figures, the answer rounds to -9 m/s .

25 **-70 m**

1. The acceleration due to gravity is g .
2. Use the equation $s = \mathbf{v}_o t + \frac{1}{2}at^2$.
3. Plug in the numbers: $s = (10 \text{ m/s})(5.0 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(5.0 \text{ s})^2 = -72.5 \text{ m}$. With significant figures, the answer rounds to -70 m .

26 **-100 m**

1. The acceleration due to gravity is g .
2. Use the equation $s = \mathbf{v}_o t + \frac{1}{2}at^2$.
3. Plug in the numbers: $s = (20 \text{ m/s})(7.0 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(7.0 \text{ s})^2 = -100.1 \text{ m}$. With significant figures, the answer rounds to -100 m .

27 **21 N downward**

1. Convert tension **A** into vector component notation. Use the equation $A_x = A \cos \theta$ to find the x coordinate of tension **A**: $15.0 \text{ N} \cos 135^\circ = -10.6 \text{ N}$.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of tension **A**: $15.0 \text{ N} \sin 135^\circ = 10.6 \text{ N}$. That makes tension **A** $(-10.6, 10.6) \text{ N}$ in coordinate form.
3. Convert tension **B** into components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of tension **B**: $15.0 \text{ N} \cos 45^\circ = 10.6 \text{ N}$.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of tension **B**: $15.0 \text{ N} \sin 45^\circ = 10.6 \text{ N}$. That makes tension **B** $(10.6, 10.6) \text{ N}$ in coordinate form.
5. Perform vector addition to find the net tension: $(-10.6, 10.6) \text{ N} + (10.6, 10.6) \text{ N} = (0, 21.2) \text{ N}$.
6. To counteract this tension, the tension in rope C must have a magnitude of 21.2 N and point downward. With significant figures, the magnitude is rounded to 21 N .

28 **24.7 N at 248°**

1. Convert tension **A** into vector component notation. Use the equation $A_x = A \cos \theta$ to find the x coordinate of tension **A**: $17.0 \text{ N} \cos 115^\circ = -7.18 \text{ N}$.
2. Use the equation $A_y = A \sin \theta$ to find the y coordinate of tension **A**: $17.0 \text{ N} \sin 115^\circ = 15.4 \text{ N}$. That makes tension **A** $(-7.18, 15.4) \text{ N}$ in coordinate form.
3. Convert tension **B** into components. Use the equation $B_x = B \cos \theta$ to find the x coordinate of tension **B**: $18.0 \text{ N} \cos 25^\circ = 16.3 \text{ N}$.
4. Use the equation $B_y = B \sin \theta$ to find the y coordinate of tension **B**: $18.0 \text{ N} \sin 25^\circ = 7.61 \text{ N}$. That makes tension **B** $(16.3, 7.61) \text{ N}$ in coordinate form.

5. Perform vector addition to find the net tension: $(-7.18, 15.4) \text{ N} + (16.3, 7.61) \text{ N} = (9.1, 23.0) \text{ N}$.
6. Find the angle of the tension due to **A** and **B**: $\theta = \tan^{-1}(y/x) = \tan^{-1}(2.53) = 68^\circ$.
7. Apply the equation $v = \sqrt{x^2 + y^2}$ to find the magnitude of tension due to **A** and **B**, giving you 24.7 N.
8. Find the angle of the tension in rope C. This must be opposite the tension due to **A** and **B**, which is $68^\circ + 180^\circ = 248^\circ$.

IN THIS CHAPTER

- » Calculating acceleration and speed on inclined planes
- » Figuring out the coefficients of friction
- » Factoring in friction

Chapter 5

Working with Inclined Planes

Many physics problems involve *inclined planes* — those ramps that you’re always seeing balls and carts roll down in physics classroom labs. Gravitational force is what makes carts roll down ramps, of course, but there’s more to it than that. In the classic beginning physics problem, you have to resolve the gravitational force along and perpendicular to the ramp to find the acceleration of the cart along the ramp.

In the real world, you also have friction. For example, if you’re unloading a refrigerator from a truck using a ramp and the refrigerator slides down the ramp by itself, friction happens, and you have to take that into account.

In this chapter, you’ll be guided through the wonderful world of inclined planes, providing you with plenty of practice questions to ensure that you come out a pro at handling this type of physics problem.

Breaking Ramps Up into Vectors

The first step in working with ramps of any kind is to resolve the forces that you’re dealing with, and that means using vectors. (For more on vectors, check out Chapter 3.) For example, take a look at the cart in Figure 5-1; it’s on an inclined plane, ready to roll.

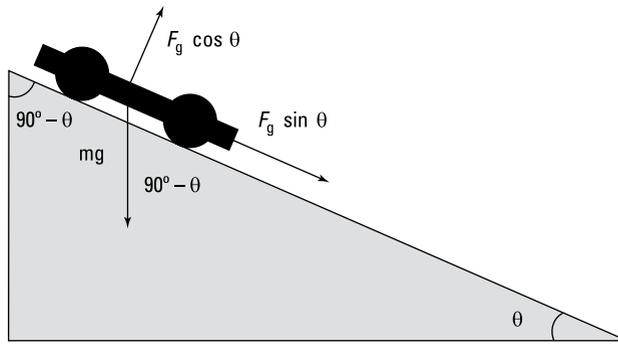


FIGURE 5-1:
A cart on an inclined plane.

© John Wiley & Sons, Inc.

The force on the cart is the force due to gravity, $F_g = mg$. So how fast will the cart accelerate along the ramp? To get the answer, you should resolve the gravitational force — not in the horizontal and vertical directions, however, but along the ramp’s inclined plane and perpendicular to that plane.



REMEMBER

The reason you resolve the gravitational force in these directions is because the force along the plane provides the cart’s acceleration while the force perpendicular to the ramp, called the *normal force*, does not. (When you start introducing friction into the picture, you’ll see that the force of friction is proportional to the normal force — that is, it’s proportional to the force with which the object going down the ramp presses against the ramp.)

Try out some questions involving force and ramps:



EXAMPLE

Q. In Figure 5-1, what are the forces along the ramp and normal to the ramp?

A. The correct answer is $F_g(\sin \theta)$ along the ramp, $F_g(\cos \theta)$ normal (perpendicular to the ramp).

- To resolve the vector F_g along the ramp, you can start by figuring out the angle between F_g and the ramp. Here’s where your knowledge of triangles comes into play. Because you know that a triangle’s angles have to add up to 180° , the angle between F_g and the ground is 90° . Figure 5-1 shows you that the ramp’s angle to the ground is θ , so you know that the angle between F_g and the ramp must be $90^\circ - \theta$.
- The angle between F_g and the ramp is $90^\circ - \theta$. So what’s the component of F_g along the ramp? Knowing the angle between F_g and the ramp, you can figure out the component of F_g along the ramp as usual:

$$\text{component of } F_g \text{ along the ramp} = F_g \cos(90^\circ - \theta)$$

- Apply the following equation:

$$\sin \theta = \cos(90^\circ - \theta)$$

$$\text{component of } F_g \text{ along the ramp} = F_g(\cos[90^\circ - \theta]) = F_g(\sin \theta) \text{ along the ramp}$$

Notice that this makes sense because as θ goes to 0° , the force along the ramp also goes to zero, and as θ goes to 90° , the force along the ramp goes to F_g .

4. Solve for the normal force, F_n , perpendicular to the ramp: $F_g \cos \theta$. The normal force, F_n , must exactly balance the component of the force of gravity perpendicular to the inclined plane. Apply the following equation:

$$\cos \theta = \sin (90^\circ - \theta)$$

$$F_n = \text{force perpendicular to the ramp} = F_g (\sin [90^\circ - \theta]) = F_g (\cos \theta)$$

- 1 Suppose that the cart in Figure 5-1 has a mass of 1.0 kg and the angle $\theta = 30^\circ$. What are the forces on the cart along and normal to the ramp?

- 2 Suppose that the cart in Figure 5-1 has a mass of 3.0 kg and the angle $\theta = 45^\circ$. What are the forces on the cart along and normal to the ramp?

- 3 You have a block of ice with a mass of 10.0 kg on a ramp at an angle of 23° . What are the forces on the ice along and normal to the ramp?

- 4 You have a refrigerator with a mass of 1.00×10^2 kg on a ramp at an angle of 19° . What are the forces on the refrigerator along and normal to the ramp?

Acceleration and Inclined Planes

When you have a block of ice (read: frictionless) moving down a ramp, it's being acted on by forces, which means that it's accelerated. How fast is it being accelerated? When you know that $F = ma$, you can solve for the acceleration.

After you solve for the force along the ramp, you can get the acceleration ($a = F/m$) along the ramp. Your block of ice is going to accelerate down the ramp.



Q. Suppose you have a block of ice on a ramp at 40° , and it slides down. What is its acceleration?

EXAMPLE

A. The correct answer is 6.3 m/s^2 .

1. What's important here is the force along the ramp: $F_g \sin \theta = mg(\sin \theta)$.
2. The acceleration of the ice is $F/m = mg(\sin \theta)/m = g(\sin \theta)$. In other words, the acceleration is the component of g acting along the ramp. Note that this result is independent of mass.
3. Plug in the numbers: $g(\sin \theta) = 6.3 \text{ m/s}^2$.

5 Suppose that a block of ice is on a ramp with an angle of 60° . What is its acceleration?

6 You're unloading a couch on a cart from a moving van. The couch gets away from you on the 27° ramp. Neglecting friction, what is its acceleration?

7 You have a block of ice with a mass of 10.0 kg on a ramp with an angle of 23° when it slips away from you. What is its acceleration down the ramp?

8 You're sliding down a toboggan run at 35° . What is your acceleration?

Running Down Ramps: Speed

When objects slide (frictionlessly) down a ramp, they're acted on by a force, which means that they're accelerated and therefore their speed changes. The equation to use in physics problems like these is

$$v_f^2 - v_o^2 = 2as$$



REMEMBER

Finding the object's final speed under these circumstances is easy when you remember that $a = g(\sin \theta)$, s is the length of the ramp, and v_o is usually 0.



EXAMPLE

Q. Say you have a block of ice on a ramp at 20° , and it slides down a ramp of 5.0 meters. What is its final speed at the bottom of the ramp?

A. The correct answer is 5.8 m/s.

1. The force along the ramp is $F_g(\sin \theta) = mg(\sin \theta)$.
2. The acceleration of the ice is $F / m = mg(\sin \theta) / m = g(\sin \theta)$.
3. Use the equation $v_f^2 = 2as = 2gs(\sin \theta)$. Plug in the numbers: $v_f^2 = 34 \text{ m}^2/\text{s}^2$, which means $v_f = 5.8 \text{ m/s}$.

9 Starting from rest, you go down a 100-m ski jump of 60° . What is your speed at takeoff?

10 You're heading down a toboggan run of 1 km at an angle of 18° . What is your final speed?

11 You have a block of ice on a ramp with an angle of 23° when it slips away from you. What is its speed at the bottom of the 6.0-m ramp?

12 A cart starts at the top of a 50-m slope at an angle 38° . What is the cart's speed at the bottom?

Friction on Inclined Planes

In the real world, when things slide down ramps, friction is involved, and the force of friction opposes the motion down the ramp. The force of friction is proportional to the force driving the two surfaces together; the stronger that force, the more friction is involved.

When something goes down a ramp, the force driving the two surfaces together is the normal force because it's the force perpendicular to the ramp's surface. The constant of proportionality is something you have to measure yourself; if the ramp is made of steel, there's a different amount of friction than if it's made of sandpaper. In the equation relating the normal force to the force of friction, F_f , the constant μ is called the *coefficient of friction* (a positive dimensionless number that's usually less than 1.0):

$$F_f = \mu F_n$$

Following are a couple of questions to test your understanding of the coefficient of friction:



EXAMPLE

Q. You're pushing a refrigerator along your kitchen floor and need to apply 100 N to get it moving. If your refrigerator has a mass of 100 kg, what is the coefficient of friction?

A. The correct answer is 0.1.

1. The force due to friction is $F_f = \mu F_n$, so $\mu = F_f / F_n$.
2. The force due to friction, F_f , is 100 N, and the normal force, F_n , is $(100)g = 980$ N.
3. Use the equation $\mu = F_f / F_n$ to get the coefficient of friction (note that μ has no units): $100 / 980 = 0.1$.

13 You're pushing a 15-kg box of books across the carpet and need to apply 100 N. What is the coefficient of friction?

14 You're pushing a 70-kg easy chair from one room to the next. If you need to apply 200 N, what is the coefficient of friction?

Starting from zero: Static friction

Two coefficients of friction correspond to two different physical processes. The first, called the *coefficient of static friction*, applies when you start pushing something at rest to get it moving. When you already have something moving and need to keep applying a force to keep it in motion, that's called the *coefficient of kinetic friction* (see the next section for more).



The coefficient of static friction, μ_s , is usually larger than the coefficient of kinetic friction, μ_k , and both are always greater than 0.0 and almost always less than 1.0.

REMEMBER



Q. Suppose that you need to move a 100-kg desk. If the coefficient of static friction between the floor and the desk is 0.2, how much force do you have to apply to get the desk to start to move?

EXAMPLE

A. The correct answer is 200 N.

1. The force of friction here is $F = \mu_s F_n = \mu_s mg$.
2. Plug in the numbers: $(0.2)(100 \text{ kg})(9.8 \text{ m/s}^2) = 196 \text{ N}$. From significant figures, this answer rounds to 200 N.

15 You're standing at the top of a ski slope and need 15 N of force to get yourself moving. If your mass is 60 kg, what is the coefficient of static friction, μ_s ?

16 You've started to pull a garbage can out to the curb. If the can has a mass of 20 kg and you need to apply 70 N to get the can moving, what is the coefficient of static friction, μ_s ?

Already in motion: Kinetic friction

The second kind of friction is *kinetic friction*, which is usually less than the force of friction you need to overcome static friction. Kinetic friction occurs when an object is already in motion.

Static friction is the force that must be overcome to get something to move, and it's usually larger than kinetic friction, the force that pushes against a moving object. The equation for kinetic friction is $F = \mu_k F_n$.



EXAMPLE

Q. Suppose you have a 5.0-kg block of ice, and it takes 5 N to keep it moving across the floor. What is the coefficient of kinetic friction, μ_k ?

A. The correct answer is 0.1.

1. The force of kinetic friction is $F_k = \mu_k F_g = \mu_k mg$.
2. Solve for μ_k : $\mu_k = F_k / (mg)$.
3. Plug in the numbers: $\mu_k = 5 \text{ N} / (5 \text{ kg} \times 9.8 \text{ m/s}^2) = 0.1$.

17 You're cross-country skiing and need 20 N of force to keep going. If you have a mass of 70 kg, what is the coefficient of kinetic friction, μ_k ?

18 You're skating and need 17 N to keep going. If you have a mass of 80 kg, what is the coefficient of kinetic friction, μ_k ?

Static Friction along Ramps

Figure 5-2 shows a box on a ramp. Suppose that the box contains a new flat-screen TV that you're pushing up the ramp and into your house.

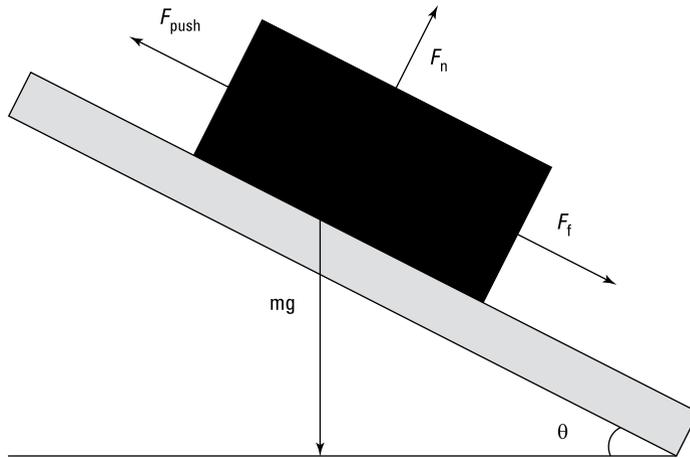


FIGURE 5-2:
An object on a
ramp.

© John Wiley & Sons, Inc.

A number of forces are acting on the box, including both gravity and friction, and you need to take both into account. There's also the force exerted upon the box as you push it up the ramp. So how do you balance all the forces? How much force is needed to get the box moving up the ramp into the house?

To figure that out, you need to calculate the forces along the ramp. There's the component of the gravitational force along the ramp, which is $mg(\sin \theta)$. But what is the force due to static friction along the ramp?

To find that, you use the equation $F_s = \mu_s F_n$. What is the normal force, F_n ? You already know the answer — it's $F_n = mg(\cos \theta)$. That makes the force due to static friction $\mu_s mg(\cos \theta)$.

The force of static friction points along the ramp, and you've calculated the force due to gravity along the ramp. Both of these forces point down the ramp and need to be overcome by the force pushing up the ramp. So in other words, F_{push} is

$$F_{push} = F_g(\sin \theta) + F_s$$

Where $F_g(\sin \theta)$ is the force due to gravity acting along the ramp, and F_s is the force due to static friction. Plugging in these forces gives you:

$$F_{push} = F_g(\sin \theta) + F_s = mg(\sin \theta) + \mu_s mg(\cos \theta)$$



EXAMPLE

Q. Suppose that the flat-screen TV's box has a mass of 1.00×10^2 kg, and the ramp has an angle of 23° . What is the force needed to get the box moving up the ramp if the coefficient of static friction is 0.20?

A. The correct answer is 563 N.

1. The equation to use is $F_{push} = mg(\sin \theta) + \mu_s mg(\cos \theta)$

2. Plug in the numbers: $F_{push} = 383 \text{ N} + 180 \text{ N} = 563 \text{ N}$.

19 You're dragging your little brother up the 25° wheelchair ramp at the doctor's office. If he has a mass of 40.0 kg and the coefficient of static friction, μ_s , is 0.15, how much force will you need to apply to get him moving?

20 Suppose that you're struggling to get a 20.0-kg block of ice moving up a 40.0° ramp. If the coefficient of static friction, μ_s , is a low 0.050, how much force will you need to apply to overcome the weight pulling the block down the ramp and static friction?

Kinetic Friction along Ramps

Two kinds of friction — static and kinetic — mean that you also have to handle ramp problems where kinetic friction is involved, as when an object on a ramp is sliding *down* that ramp. Because the object is moving, kinetic friction applies. That means you can solve problems with kinetic friction as easily as those that involve static friction.

Here's the equation for the force needed to get an object moving and thus overcome static friction:

$$F = F_g(\sin \theta) + F_s = mg(\sin \theta) + \mu_s mg(\cos \theta)$$



REMEMBER

To convert this equation to using kinetic friction, all you have to do is to change from using the coefficient of static friction, μ_s , to using the coefficient of kinetic friction, μ_k :

$$F = F_g(\sin \theta) + F_k = mg(\sin \theta) + \mu_k mg(\cos \theta)$$

That's all there is to it.



Q. Suppose that you're pushing a 40.0-kg plastic crate up a 19.0° ramp. The coefficient of kinetic friction, μ_k , is 0.100, and you need to apply a force to keep the crate moving. What is the force you will need to apply?

EXAMPLE

A. The correct answer is 165 N.

1. The equation here is $F = F_g(\sin \theta) + F_k = mg(\sin \theta) + \mu_k mg(\cos \theta)$.
2. Putting in the numbers gives you $F = 128 \text{ N} + 37 \text{ N} = 165 \text{ N}$.

21 You're dragging your little sister up the 25° wheelchair ramp at the doctor's office. If she has a mass of 30.0 kg and the coefficient of kinetic friction, μ_k , is 0.10, how much force will you need to apply to keep her going?

22 You're pushing a box of books with a mass of 25 kg up a 40° ramp. If the coefficient of kinetic friction, μ_k , is 0.27, how much force will you need to apply to keep the box moving up the ramp?

23 You're pushing a chest of drawers up an 18° ramp. If it has a mass of 50.0 kg and the coefficient of kinetic friction, μ_k , is 0.20, how much force will you need to keep it moving?

24 You want to keep a 120-kg refrigerator moving up a 23° ramp. If the coefficient of kinetic friction, μ_k , is 0.20, how much force will you need to keep it moving?

Acceleration along Ramps Including Friction

Suppose that you have a crate that has been mistakenly placed near the top of a long ramp, and it starts sliding down that ramp. The preceding section helps you calculate how much force acts on that crate, so how about calculating its acceleration down the incline?

The object is sliding down the ramp — you're not pushing it — which means the force of kinetic friction is opposing (not adding to) the component of gravity along the ramp. So the net force on the crate is

$$F = F_g(\sin \theta) - F_k = mg(\sin \theta) - \mu_k mg(\cos \theta)$$

Because $a = F / m$, the acceleration of the crate is

$$a = g(\sin \theta) - \mu_k g(\cos \theta)$$



Q. A plastic crate slips down a 19° ramp with a coefficient of kinetic friction, μ_k , of 0.10. What is its acceleration as it slides?

EXAMPLE

A. The correct answer is 2.3 m/s^2 .

1. You can use the equation $a = g(\sin \theta) - \mu_k g(\cos \theta)$.
2. Plug in the numbers: $a = 3.2 \text{ m/s}^2 - 0.93 \text{ m/s}^2 = 2.3 \text{ m/s}^2$.

25 You're dragging a suitcase up a ramp into a luxury hotel when it gets away from you. If the angle of the ramp is 31° and the coefficient of kinetic friction is 0.1, what is the suitcase's acceleration down the ramp?

26 You drop a 5.0-kg box on a ramp of 12° , and the coefficient of kinetic friction is 0.15. Will the box slide down the ramp?

27 You drop a 1.0-kg book on a 15° ramp, and the coefficient of kinetic friction is 0.30. Will the book slide down the ramp?

28 A refrigerator breaks away from the movers and slides down a 23° ramp that has a coefficient of kinetic friction of 0.25. What is its acceleration?

Answers to Problems about Inclined Planes

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 4.9 N along the ramp, 8.5 N normal to the ramp

1. You know that the forces on the cart are $F_g \sin \theta$ along the ramp and $F_g \cos \theta$ normal to the ramp.
2. Plug the numbers into $F_g = mg$: $(1.0 \text{ kg})(9.8 \text{ m/s}^2) = 9.8 \text{ N}$.
3. The force along the ramp is $F_g(\sin \theta) = (9.8 \text{ N})(\sin 30^\circ) = 4.9 \text{ N}$.
4. The force normal to the ramp is $F_g(\cos \theta) = (9.8 \text{ N})(\cos 30^\circ) = 8.5 \text{ N}$.

2 21 N along the ramp, 21 N normal to the ramp

1. The forces on the cart are $F_g(\sin \theta)$ along the ramp and $F_g(\cos \theta)$ normal to the ramp.
2. Plug the numbers into $F_g = mg$: $(3.0 \text{ kg})(9.8 \text{ m/s}^2) = 29 \text{ N}$.
3. The force along the ramp is $F_g(\sin \theta) = (29 \text{ N})(\sin 45^\circ) = 21 \text{ N}$.
4. The force normal to the ramp is $F_g(\cos \theta) = (29 \text{ N})(\cos 45^\circ) = 21 \text{ N}$.

3 38 N along the ramp, 90 N normal to the ramp

1. The forces on the ice are $F_g(\sin \theta)$ along the ramp and $F_g(\cos \theta)$ normal to the ramp.
2. Plug the numbers into $F_g = mg$: $(10.0 \text{ kg})(9.8 \text{ m/s}^2) = 98 \text{ N}$.
3. The force along the ramp is $F_g(\sin \theta) = (98 \text{ N})(\sin 23^\circ) = 38 \text{ N}$.
4. The force normal to the ramp is $F_g(\cos \theta) = (98 \text{ N})(\cos 23^\circ) = 90 \text{ N}$.

4 320 N along the ramp, 930 N normal to the ramp

1. The forces on the ice are $F_g(\sin \theta)$ along the ramp and $F_g(\cos \theta)$ normal to the ramp.
2. Plug the numbers into $F_g = mg$: $(1.00 \times 10^2 \text{ kg})(9.8 \text{ m/s}^2) = 980 \text{ N}$.
3. The force along the ramp is $F_g(\sin \theta) = (980 \text{ N})(\sin 19^\circ) = 320 \text{ N}$.
4. The force normal to the ramp is $F_g(\cos \theta) = (980 \text{ N})(\cos 19^\circ) = 930 \text{ N}$.

5 8.5 m/s²

1. The force along the ramp is $F_g(\sin \theta) = mg(\sin \theta)$.
2. The acceleration of the ice is $F / m = mg(\sin \theta) / m = g(\sin \theta)$.
3. Plugging in the numbers gives you $g(\sin \theta) = 8.5 \text{ m/s}^2$.

6 4.4 m/s²

1. The force along the ramp is $F_g(\sin \theta) = mg(\sin \theta)$.
2. The acceleration of the cart is $F / m = mg(\sin \theta) / m = g(\sin \theta)$.
3. Plugging in the numbers gives you $g(\sin \theta) = 4.4 \text{ m/s}^2$.

7 **3.8 m/s²**

1. The force along the ramp is $F_g(\sin \theta) = mg(\sin \theta)$.
2. The acceleration of the ice is $F / m = mg(\sin \theta) / m = g(\sin \theta)$.
3. Plugging in the numbers gives you $g(\sin \theta) = 3.8 \text{ m/s}^2$.

8 **5.6 m/s²**

1. The force along the ramp is $F_g(\sin \theta) = mg(\sin \theta)$.
2. Your acceleration is $F / m = mg(\sin \theta) / m = g(\sin \theta)$.
3. Plugging in the numbers gives you $g(\sin \theta) = 5.6 \text{ m/s}^2$.

9 **40 m/s**

1. The force along the ramp is $F_g(\sin \theta) = mg(\sin \theta)$.
2. Your acceleration is $F / m = mg(\sin \theta) / m = g(\sin \theta)$.
3. Use the equation $v_f^2 = 2as = 2gs(\sin \theta)$.
4. Solving for $v_f = 41 \text{ m/s}$. Round to 40 m/s to account for significant figures.

10 **80 m/s**

1. The force along the ramp is $F_g(\sin \theta) = mg(\sin \theta)$.
2. Your acceleration is $F / m = mg(\sin \theta) / m = g(\sin \theta)$.
3. Use the equation $v_f^2 = 2as = 2gs(\sin \theta)$.
4. Solving for $v_f = 78 \text{ m/s}$. Round to 80 m/s to account for significant figures.

11 **6.8 m/s**

1. The force along the ramp is $F_g(\sin \theta) = mg(\sin \theta)$.
2. The block of ice's acceleration is $F / m = mg(\sin \theta) / m = g(\sin \theta)$.
3. Use the equation $v_f^2 = 2as = 2gs(\sin \theta)$.
4. Solving for $v_f = 6.8 \text{ m/s}$.

12 **25 m/s**

1. The force along the ramp is $F_g(\sin \theta) = mg(\sin \theta)$.
2. The cart's acceleration is $F / m = mg(\sin \theta) / m = g(\sin \theta)$.
3. Use the equation $v_f^2 = 2as = 2gs(\sin \theta)$.
4. Solving for $v_f = 25 \text{ m/s}$.

13 **0.7**

1. Solve for μ . The force due to friction is $F_f = \mu F_n$, so $\mu = F_f / F_n$.
2. The force due to friction, F_f , is 100 N, and the normal force, F_n , is $(15 \text{ kg})g = 148 \text{ N}$.
3. Use the equation $\mu = F_f / F_n$ to get the coefficient of friction (note that μ has no units): $100 / 148 = 0.68$. Round to 0.7 to account for significant figures.

14 **0.3**

1. Solve for μ . The force due to friction is $F_f = \mu F_n$, so $\mu = F_f / F_n$.
2. The force due to friction, F_f , is 200 N, and the normal force, F_n , is $(70 \text{ kg})g = 690 \text{ N}$.
3. Use the equation $\mu = F_f / F_n$ to get the coefficient of friction: $200 / 690 = 0.29$. Round to 0.3 to account for significant figures.

15 **0.03**

1. Solve for μ_s . The force due to friction is $F_f = \mu_s F_n$, so $\mu_s = F_f / F_n$.
2. The force due to friction, F_f , is 15 N, and the normal force, F_n , is $(60 \text{ kg})g = 590 \text{ N}$.
3. Use the equation $\mu_s = F_f / F_n$ to get the coefficient of static friction: $15 / 590 = 0.03$.

16 **0.4**

1. Solve for μ_s . The force due to friction is $F_f = \mu_s F_n$, so $\mu_s = F_f / F_n$.
2. The force due to friction, F_f , is 70 N, and the normal force, F_n , is $(20 \text{ kg})g = 196 \text{ N}$.
3. Use the equation $\mu_s = F_f / F_n$ to get the coefficient of static friction: $70 / 196 = 0.36$. From significant figures, this answer rounds to 0.4.

17 **0.03**

1. Solve for μ_k . The force due to friction is $F_f = \mu_k F_n$, so $\mu_k = F_f / F_n$.
2. The force due to friction that you have to overcome, F_f , is 20 N, and the normal force, F_n , is $(70 \text{ kg})g = 690 \text{ N}$.
3. Use the equation $\mu_k = F_f / F_n$ to get the coefficient of kinetic friction: $20 / 690 = 0.03$.

18 **0.02**

1. Solve for μ_k . The force due to friction is $F_f = \mu_k F_n$, so $\mu_k = F_f / F_n$.
2. The force due to friction that you have to overcome, F_f , is 17 N, and the normal force, F_n , is $(80 \text{ kg})g = 780 \text{ N}$.
3. Use the equation $\mu_k = F_f / F_n$ to get the coefficient of kinetic friction: $17 / 780 = 0.02$.

19 **220 N**

1. Calculate the forces you need to overcome: The force due to gravity is $mg(\sin \theta)$; and the force due to friction, F_f , is $F = \mu_s F_n$. You need to find the normal force.
2. The equation for normal force is $F_n = mg(\cos \theta)$. Use the normal force to calculate the force due to friction: $\mu_s mg(\cos \theta)$.
3. The total force you have to overcome is $F = mg(\sin \theta) + \mu_s mg(\cos \theta)$.
4. Plug in the numbers: $F = mg(\sin \theta) + \mu_s mg(\cos \theta) = 166 \text{ N} + 53 \text{ N} = 219 \text{ N}$. The final answer should be rounded to 220 N. Note that the value for g is only known to two significant figures (9.8) because it varies in the third digit depending on location on Earth.

20 **130 N**

1. Calculate the forces you need to overcome: The force due to gravity is $mg(\sin \theta)$; and the force due to friction, F_f , is $F = \mu_s F_n$. You need to find the normal force.
2. The equation for normal force is $F_n = mg(\cos \theta)$. Use the normal force to calculate the force due to friction: $\mu_s mg(\cos \theta)$.
3. The total force you have to overcome is $F = mg(\sin \theta) + \mu_s mg(\cos \theta)$.
4. Plug in the numbers: $F = mg(\sin \theta) + \mu_s mg(\cos \theta) = 126 \text{ N} + 7.5 \text{ N} = 133 \text{ N}$. Note that most of this force is due to the component of the weight along the ramp. The final answer should be rounded to 130 N.

21 **150 N**

1. Determine the forces you need to overcome: The force due to gravity is $mg(\sin \theta)$; and the force due to kinetic friction is $F = \mu_k F_n$. You need to find the normal force.
2. The equation for normal force is $F_n = mg(\cos \theta)$. Use the normal force to calculate the force due to kinetic friction: $\mu_k mg(\cos \theta)$.
3. The force you have to overcome is $F = mg(\sin \theta) + \mu_k mg(\cos \theta)$.
4. Plug in the numbers: $F = mg(\sin \theta) + \mu_k mg(\cos \theta) = 124 \text{ N} + 27 \text{ N} = 150 \text{ N}$.

22 **210 N**

1. Calculate the forces you need to overcome: The force due to gravity is $mg(\sin \theta)$; and the force due to kinetic friction is $F = \mu_k F_n$. You need to find the normal force.
2. The equation for normal force is $F_n = mg(\cos \theta)$. Use the normal force to calculate the force due to kinetic friction: $\mu_k mg(\cos \theta)$.
3. The force you have to overcome to keep the box moving is $F = mg(\sin \theta) + \mu_k mg(\cos \theta)$.
4. Plug in the numbers: $F = mg(\sin \theta) + \mu_k mg(\cos \theta) = 160 \text{ N} + 50 \text{ N} = 210 \text{ N}$.

23 **240 N**

1. Calculate the forces you need to overcome: The force due to gravity is $mg(\sin \theta)$; and the force due to kinetic friction is $F = \mu_k F_n$. You need to find the normal force.
2. The equation for normal force is $F_n = mg(\cos \theta)$. Use the normal force to calculate the force due to kinetic friction: $\mu_k mg(\cos \theta)$.
3. The force you have to overcome to keep the chest of drawers moving is $F = mg(\sin \theta) + \mu_k mg(\cos \theta)$.
4. Plug in the numbers: $F = mg(\sin \theta) + \mu_k mg(\cos \theta) = 150 \text{ N} + 90 \text{ N} = 240 \text{ N}$.

24 **680 N**

1. Calculate the forces you need to overcome: The force due to gravity is $mg(\sin \theta)$; and the force due to kinetic friction is $F = \mu_k F_n$. You need to find the normal force.
2. The equation for normal force is $F_n = mg(\cos \theta)$. Use the normal force to calculate the force due to kinetic friction: $\mu_k mg(\cos \theta)$.
3. The force you have to overcome to keep the refrigerator moving is $F = mg(\sin \theta) + \mu_k mg(\cos \theta)$.
4. Plug in the numbers: $F = mg(\sin \theta) + \mu_k mg(\cos \theta) = 460 \text{ N} + 220 \text{ N} = 680 \text{ N}$.

25 4 m/s^2

1. Calculate the forces on the suitcase: The force due to gravity is $mg(\sin \theta)$; and the force due to kinetic friction is $F = \mu_k F_n$. You need to find the normal force.
2. The equation for the normal force is $F_n = mg(\cos \theta)$. Use the normal force to calculate the force due to kinetic friction: $\mu_k mg(\cos \theta)$.
3. The net force on the suitcase along the ramp is $F = mg(\sin \theta) - \mu_k mg(\cos \theta)$.
4. Divide the net force along the ramp by m to get the acceleration: $a = g(\sin \theta) - \mu_k g(\cos \theta)$.
5. Plug in the numbers: $a = g(\sin \theta) - \mu_k g(\cos \theta) = 5.0 \text{ m/s}^2 - 0.8 \text{ m/s}^2 = 4.2 \text{ m/s}^2$. Round to 4 m/s^2 to account for significant figures.

26 **Yes, the box will slide.**

1. Calculate the forces on the box, and if the force down the ramp is larger than the force up the ramp, the box will slide. The force down the ramp is $mg(\sin \theta)$.
2. The force due to kinetic friction pointing up the ramp is $F = \mu_k F_n$, which means you need to find the normal force.
3. The equation for the normal force is $F_n = mg(\cos \theta)$. Use the normal force to calculate the force due to kinetic friction: $\mu_k mg(\cos \theta)$.
4. The net force on the suitcase down the ramp is $F = mg(\sin \theta) - \mu_k mg(\cos \theta)$.
5. Plug in the numbers $F = mg(\sin \theta) - \mu_k mg(\cos \theta) = 10 \text{ m/s}^2 - 7.2 \text{ m/s}^2 = 2.8 \text{ N}$ down the ramp, so the box will slide.

27 **No, the book will not slide.**

1. Calculate the forces on the book, and if the force down the ramp is larger than the force up the ramp, the book will slide. The force down the ramp is $mg(\sin \theta)$.
2. The force due to kinetic friction pointing up the ramp is $F = \mu_k F_n$, which means you need to find the normal force.
3. The equation for the normal force is $F_n = mg(\cos \theta)$. Use the normal force to calculate the force due to kinetic friction: $\mu_k mg(\cos \theta)$.
4. The net force on the book down the ramp is $F = mg(\sin \theta) - \mu_k mg(\cos \theta)$.
5. Plug in the numbers $F = mg(\sin \theta) - \mu_k mg(\cos \theta) = 2.5 \text{ N} - 2.8 \text{ N} = -0.3 \text{ N}$ along the ramp, so the book will not slide.

28 1.5 m/s^2 down the ramp

1. Calculate the forces on the refrigerator: The force down the ramp is $mg(\sin \theta)$; and the force due to kinetic friction pointing up the ramp is $F = \mu_k F_n$. You need to find the normal force.
2. The equation for the normal force is $F_n = mg(\cos \theta)$. Use the normal force to calculate the force due to kinetic friction: $\mu_k mg(\cos \theta)$.
3. The net force on the refrigerator down the ramp is $F = mg(\sin \theta) - \mu_k mg(\cos \theta)$.
4. Divide by m to get the acceleration: $a = g \sin \theta - \mu_k g \cos \theta$.
5. Plug in the numbers: $a = g(\sin \theta) - \mu_k g(\cos \theta) = 3.8 \text{ m/s}^2 - 2.3 \text{ m/s}^2 = 1.5 \text{ m/s}^2$.

- » Converting angles
- » Handling period and frequency
- » Working with angular frequency
- » Using angular acceleration

Chapter 6

Round and Round: Circular Motion

In physics, motion in circles is just as important as motion along lines, but there are all kinds of differences between the two. With circular motion, you no longer work with meters but instead work with *radians*, which are angular units. Velocity in circular motion is no longer in meters/second but rather radians/second.

The bottom line is that circular motion requires a whole new way of thinking. And the best way to get a handle on that switch is to break it down and work through some problems. That's what this chapter is about — solving problems involving angular motion, angular velocity, and angular acceleration.

Converting between Angles

The first step in working with angular motion is to know how to measure that motion. You use radians, not meters, and you have to know what that means.

Take a look at Figure 6-1, where you can see an angle, θ , in a circle. Suppose that θ is 45° . What is that in radians?

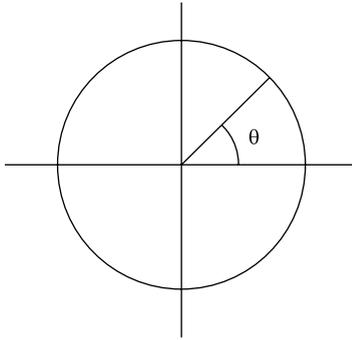


FIGURE 6-1:
An angle
in a circle.

© John Wiley & Sons, Inc.

There are 2π radians in a full, 360° circle. That means that to convert 45° from degrees to radians, you multiply by $2\pi/360^\circ$, like so:

$$45^\circ \left(\frac{2\pi}{360^\circ} \right) = \frac{\pi}{4}$$



REMEMBER

Angles measured in radians don't have units! That's because they're expressed as a fraction of 2π . You'll sometimes see angles expressed as though radians were units, but technically, they're not.

The conversion factor, $2\pi/360^\circ$, is usually written as $\pi/180^\circ$, which makes this conversion:

$$45^\circ \left(\frac{\pi}{180^\circ} \right) = \frac{\pi}{4}$$

Conversely, to convert from radians to degrees, you multiply by $180^\circ/\pi$. For example, the equation to convert $\pi/4$ to an angle looks like this:

$$\frac{\pi}{4} \cdot \frac{180^\circ}{\pi} = 45^\circ$$



EXAMPLE

Q. Convert 180° into radians.

A. The correct answer is π .

1. Use the conversion factor $\pi/180^\circ$.
2. Plug in the numbers:

$$180^\circ \left(\frac{\pi}{180^\circ} \right) = \pi$$

1 What is 23° in radians?

2 What is $\pi/16$ in degrees?

Period and Frequency

When describing the way things go in circles, you don't just use radians; you also can specify the time it takes. The time it takes for an object to complete an orbit is referred to as the *period* of its motion. For example, if the object is traveling at speed v , then the time it takes to go around the circle — the distance it travels in the circle's circumference, $2\pi r$ — will be

$$T = \frac{2\pi r}{v}$$

Note the symbol of the radius of a circle: r . That's half the circle's diameter, which is d . So $r = d/2$. Note also the symbol for the period: T . With this equation, given an orbiting object's speed and the radius of the circle, you can calculate the object's period.

Another time measurement you'll see in physics problems is *frequency*. Whereas the period is the time an object takes to go around in a circle, the frequency is the number of circles the object makes per second. The frequency, f , is connected to the period like this:

$$f = \frac{1}{T}$$



Q. The moon's orbital radius is 3.85×10^8 m, and its period is about 27.3 days. What is its speed as it goes around Earth?

EXAMPLE

A. The correct answer is 1,020 m/s, when rounded for significant figures.

1. Convert 27.3 days to seconds:

$$\begin{aligned} 27.3 \text{ days} &\times \frac{24 \text{ hours}}{\text{day}} \times \frac{60 \text{ minutes}}{\text{hour}} \\ &\times \frac{60 \text{ seconds}}{\text{minute}} = 2.36 \times 10^6 \text{ s} \end{aligned}$$

2. Use the equation for the period to solve for speed:

$$v = \frac{2\pi r}{T}$$

3. Plug in the numbers:

$$v = \frac{2\pi r}{T} = \frac{2\pi(3.85 \times 10^8 \text{ m})}{2.36 \times 10^6 \text{ s}} = 1,024 \text{ m/s}$$

3 You have a ball on a string, and you're whipping it around in a circle. If the radius of its circle is 1.0 m and its period is 1.0 s, what is its speed?

4 You have a toy plane on a wire, and it's traveling around in a circle. If the radius of its circle is 10.0 m and its period is 0.75 s, what is its speed?

Getting into Angular Velocity

There are analogs of every linear motion quantity (such as distance, velocity, and acceleration) in angular motion, and that's one of the things that makes angular motion easier to work with. The velocity of an object in linear motion is shown in the following equation (this is actually a vector equation, of course, but the vector nature of angular motion isn't discussed until Chapter 10, so this equation is looked at in scalar terms):

$$v = \frac{\Delta s}{\Delta t}$$

What's the analog of this equation in angular terms? That's easy; you just substitute angle θ for the distance, so the angular velocity is θ/t . That means that angular velocity is the angle (in radians) that an object sweeps through per second. The symbol for angular velocity is ω , so you can write the equation for angular velocity this way:

$$\omega = \frac{\Delta \theta}{\Delta t}$$

Figure 6-2 shows a line sweeping around in a circle. At a particular moment, it's at angle θ , and if it took time t to get there, its angular velocity is $\omega = \theta/t$.

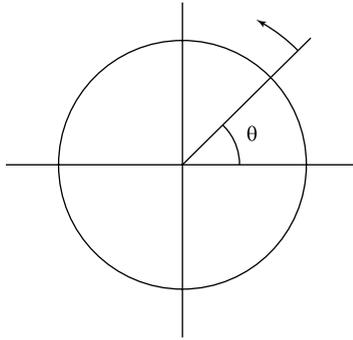


FIGURE 6-2:
Angular
velocity in a
circle.

© John Wiley & Sons, Inc.

So if the line in Figure 6-2 completes a full circle in 1.0 s, its angular velocity is $2\pi/1.0\text{ s} = 2\pi$ radians/s (because there are 2π radians in a complete circle). Technically speaking, radian isn't a physical unit of measure (it's a ratio), so the angular velocity can also be written $2\pi\text{ s}^{-1}$.

Given the angular velocity, you also can find the angle swept through in a number of seconds:

$$\Delta\theta = \omega \cdot \Delta t$$



EXAMPLE

Q. The moon goes around Earth in about 27.3 days. What is its angular velocity?

A. The correct answer is 2.66×10^{-6} radians/s.

1. Convert 27.3 days to seconds:

$$27.3 \text{ days} \times \frac{24 \text{ hours}}{\text{day}} \times \frac{60 \text{ minutes}}{\text{hour}} \times \frac{60 \text{ seconds}}{\text{minute}} = 2.36 \times 10^6 \text{ s}$$

2. Use the equation for angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

3. Plug in the numbers:

$$\begin{aligned} \omega = \frac{\Delta\theta}{\Delta t} &= \frac{2\pi}{2.36 \times 10^6 \text{ s}} \\ &= 2.66 \times 10^{-6} \text{ radians/s} \end{aligned}$$

5 You have a toy plane on a string that goes around three complete circles in 9.0 s. What is its angular velocity?

6 You're swinging a baseball bat around, getting ready for your shot at the ball. If you swing the bat in a half circle in 1.0 s, what is its angular velocity?

7 A satellite is orbiting Earth at 8.7×10^{-4} radians/s. How long will it take to circle the entire world?

8 A merry-go-round is spinning around at 2.1 radians/s. How long will it take to go in a complete circle?

Whipping Around with Angular Acceleration

Just as with linear motion, you can have acceleration when you're dealing with angular motion. For example, the line in Figure 6-2 may be sweeping around the circle faster and faster, which means it's accelerating.

In linear motion, the following is the equation for *acceleration*, the rate at which the object's velocity is changing:

$$a = \frac{\Delta v}{\Delta t}$$

As with all the equations of motion, you need only to substitute the correct angular quantities for the linear ones. In this case, v becomes ω . So the angular acceleration is $\Delta\omega/\Delta t$.

The symbol for linear acceleration is a , and the symbol for angular acceleration is α , which makes the equation for angular acceleration:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

The unit for angular acceleration is radians/s² (or, technically, just s⁻²).



Q. Your toy plane on a string accelerates from $\omega = 2.1$ radians/s to 3.1 radians/s in 1.0 s. What is its angular acceleration?

EXAMPLE

A. The correct answer is 1.0 radian/s².

1. Use the equation for angular acceleration:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

2. Plug in the numbers:

$$\begin{aligned}\alpha = \frac{\Delta\omega}{\Delta t} &= \frac{(3.1 \text{ radians/s} - 2.1 \text{ radians/s})}{1.0 \text{ s}} \\ &= 1.0 \text{ radian/s}^2\end{aligned}$$

9 Your model globe is turning at 2.0 radians/s, which you decide isn't fast enough. So you give it a push, accelerating it in 10⁻¹ s to 5.0 radians/s. What is its angular acceleration?

10 You have a toy plane on a wire, and it's traveling in a circle at 3.5 radians/s. You speed it up to 5.4 radians/s in 3.0 s. What was its angular acceleration?

11 You're square dancing, turning your partner around at 1.0 radian/s. Then you speed up for 0.50 s at an angular acceleration of 10.0 radians/s². What is your partner's final angular speed?

12 You're trying a new yoga move, and starting your arm at rest, you accelerate it at 15 radians/s² over 1.0 s. What's your arm's final angular velocity?

Connecting Angular Velocity and Angular Acceleration to Angles

You can connect the distance traveled to the original velocity and linear acceleration like this:

$$s = v_o (t_f - t_o) + \frac{1}{2} a (t_f - t_o)^2$$

And you can make the substitution from linear to angular motion by putting in the appropriate symbols:

$$\theta = \omega_o (t_f - t_o) + \frac{1}{2} \alpha (t_f - t_o)^2$$

Using this equation, you can connect angular velocity, angular acceleration, and time to the angle.



Q. A marble is rolling around a circular track at 6.0 radians/s and then accelerates at 1.0 radian/s². How many radians has it gone through in 1 minute?

EXAMPLE

A. The correct answer is 2,200 radians.

1. Use this equation:

$$\theta = \omega_o (t_f - t_o) + \frac{1}{2} \alpha (t_f - t_o)^2$$

2. Plug in the numbers:

$$\begin{aligned} \theta &= \omega_o (t_f - t_o) + \frac{1}{2} \alpha (t_f - t_o)^2 \\ &= (6.0 \text{ radians/s})(60 \text{ s}) + \frac{1}{2} (1.0 \text{ radian/s}^2)(60 \text{ s})^2 \\ &= 360 \text{ radians} + 1,800 \text{ radians} \\ &= 2,160 \text{ radians, which rounds to 2,200 radians} \end{aligned}$$

13 Your model globe is spinning at 1.0 radian/s when you give it a push. If you accelerate it at 5.0 radian/s², how many radians has it turned through in 5.0 s?

14 Your toy plane on a wire is traveling around in a circle at 8.0 radians/s. If you accelerate it at 1.0 radian/s² for 20.0 s, how many radians has it gone through during that time?

- 15 You're whipping a ball on a string around in a circle. If it's going 7.0 radians/s and at the end of 6.0 s has gone through 60.0 radians, what was its angular acceleration?

- 16 A roulette wheel is slowing down, starting from 12.0 radians/s and going through 40.0 radians in 5.0 s. What was its angular acceleration?

Connecting Angular Acceleration and Angle to Angular Velocity

You can connect angle, angular velocity, and angular acceleration. The corresponding equation for linear motion is

$$v_f^2 - v_o^2 = 2as$$

Substituting ω for v , α for a , and θ for s gives you:

$$\omega_f^2 - \omega_o^2 = 2\alpha\theta$$

Use this equation when you want to relate angle to angular velocity and angular acceleration.



EXAMPLE

- Q.** A merry-go-round slows down from 6.5 radians/s to 2.5 radians/s, undergoing an angular acceleration of 1.0 radians/s². How many radians does the merry-go-round go through while this is happening?

- A.** The correct answer is 18 radians.

1. Start with the equation:

$$\omega_f^2 - \omega_o^2 = 2\alpha\theta$$

2. Solve for θ :

$$\theta = \frac{\omega_f^2 - \omega_o^2}{2\alpha}$$

3. Plug in the numbers:

$$\begin{aligned}\theta &= \frac{\omega_f^2 - \omega_o^2}{2\alpha} \\ &= \frac{(2.5 \text{ radians/s}^2) - (6.5 \text{ radians/s})^2}{2(-1.0 \text{ radian/s}^2)} \\ &= 18 \text{ radians}\end{aligned}$$

17 A helicopter's blades are speeding up. They go from 60 radians/s to 80 radians/s. If the angular acceleration is 10 radians/s², what is the total angle the blades have gone through?

18 Your ball on a string is traveling around in a circle. If it goes from 12 radians/s to 24 radians/s and the angular acceleration is 20 radians/s², what is the total angle the ball has gone through during this acceleration?

Handling Centripetal Acceleration

In order to keep an object going around in a circle, that object must be pulled toward the center of the circle. Take a look at the moon circling Earth in Figure 6-3. The moon is accelerated toward Earth along a radius from Earth to the moon. The acceleration needed to keep an object (in Figure 6-3, it's the moon) going around in a circle is called the *centripetal acceleration*, and it's always perpendicular to the object's travel. The centripetal acceleration points toward the center of the circle.

If you're given the linear speed of the object going in a circle and the radius of the circle, you can calculate the centripetal acceleration, a_c , like this:

$$a_c = \frac{v^2}{r}$$

This equation gives you the acceleration needed to keep an object going around in a circle. You can think of this acceleration as continually bending the direction the object is going in so it will keep going in a circle.

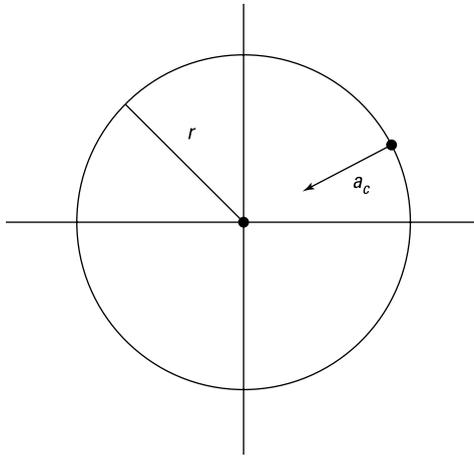


FIGURE 6-3:
Centripetal
acceleration.

© John Wiley & Sons, Inc.

The linear velocity v of an object going around in a circle is directly related to the angular velocity ω and the radius of the circle: $v = \omega r$ (for more on this, see Chapter 9). So you can also write this equation as

$$a_c = \frac{v^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r$$



Q. Given that the moon goes around Earth about every 27.3 days and that its distance from the center of Earth is 3.85×10^8 m, what is the moon's centripetal acceleration?

EXAMPLE

A. The correct answer is 2.7×10^{-3} m/s².

1. Start with this equation:

$$a_c = \frac{v^2}{r}$$

2. Find the speed of the moon. It goes $2\pi r$ in 27.3 days, so convert 27.3 to seconds:

$$27.3 \text{ days} \times \frac{24 \text{ hours}}{\text{day}} \times \frac{60 \text{ minutes}}{\text{hour}} \times \frac{60 \text{ seconds}}{\text{minute}} = 2.36 \times 10^6 \text{ s}$$

3. Therefore, the speed of the moon is

$$\frac{2\pi r}{T} = \frac{2\pi (3.85 \times 10^8 \text{ m})}{2.36 \times 10^6 \text{ s}} = 1,024 \text{ m/s}$$

4. Plug in the numbers:

$$a_c = \frac{v^2}{r} = \frac{(1,024 \text{ m/s})^2}{3.85 \times 10^8 \text{ m}} = 2.7 \times 10^{-3} \text{ m/s}^2$$

- 19 The tips of a helicopter's blades are moving at 300 m/s and have a radius of 7.0 m. What is the centripetal acceleration of those tips?

- 20 Your ball on a string is revolving around in a circle. If it's going 60 mph at a radius of 2.0 m, what is its centripetal acceleration?

Getting Forceful: Centripetal Force

To give an object moving in a circle the centripetal acceleration needed to keep moving, it needs a force applied to it. Any force that causes an object to move in a circle is a *centripetal force*. Gravity, tension, friction, and other forces can all act as centripetal forces; all of these forces can act to pull or push an object into a circle. Because $F = ma$, the centripetal force F_c is just ma_c . Here's the equation for centripetal force:

$$F_c = ma_c = \frac{mv^2}{r}$$

You can also calculate the centripetal acceleration a_c using the angular velocity: $a_c = \omega^2 r$. This means that you can also calculate the centripetal force with the following formula:

$$F_c = ma_c = m\omega^2 r$$



EXAMPLE

- Q. The moon goes around Earth about every 27.3 days with a distance from Earth of 3.85×10^8 m. If the moon's mass is 7.35×10^{22} kg, what is the centripetal force that Earth's gravity exerts on it as it orbits Earth?

A. The correct answer is 2.0×10^{20} N.

1. Start with this equation:

$$F_c = \frac{mv^2}{r}$$

2. Find the speed of the moon. It goes $2\pi r$ in 27.3 days, so convert 27.3 days to seconds:

$$27.3 \text{ days} \times \frac{24 \text{ hours}}{\text{day}} \times \frac{60 \text{ minutes}}{\text{hour}} \times \frac{60 \text{ seconds}}{\text{minute}} = 2.36 \times 10^6 \text{ s}$$

3. Therefore, the speed of the moon is

$$v = \frac{2\pi r}{T} = \frac{2\pi (3.85 \times 10^8 \text{ m})}{2.36 \times 10^6 \text{ s}} = 1,024 \text{ m/s}$$

4. Plug in the numbers:

$$\begin{aligned} F_c &= \frac{mv^2}{r} = \frac{(7.35 \times 10^{22} \text{ kg})(1,024 \text{ m/s})^2}{3.85 \times 10^8 \text{ m}} \\ &= 2.0 \times 10^{20} \text{ N} \end{aligned}$$

21 You're exerting a force on a string to keep a ball on a string going in a circle. If the ball has a mass of 0.10 kg and the angular velocity of the ball is 8.0 radians/s at a distance of 2.0 m, what is the centripetal force you need to apply to keep the ball going in a circle?

22 You have a 1.0-kg toy plane on the end of a 10-m wire, and it's going around at 6.0 radians/s. What is the force you have to apply to the wire to keep the plane going in a circle?

Answers to Problems about Circular Motion

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 0.40 radians

1. Use the conversion factor $\pi/180^\circ$.
2. Plug in the numbers:

$$23^\circ \left(\frac{\pi}{180^\circ} \right) = 0.13\pi = 0.40 \text{ radians}$$

2 11.2°

1. Use the conversion factor $180^\circ/\pi$.
2. Plug in the numbers:

$$\frac{\pi 180^\circ}{16\pi} = 11.2^\circ$$

3 6.3 m/s

1. Use the equation for the period to solve for speed:

$$v = \frac{2\pi r}{T}$$

2. Plug in the numbers:

$$v = \frac{2\pi r}{T} = \frac{2\pi(1.0 \text{ m})}{1.0 \text{ s}} = 6.3 \text{ m/s}$$

4 84 m/s

1. Use the equation for the period to solve for speed:

$$v = \frac{2\pi r}{T}$$

2. Plug in the numbers:

$$v = \frac{2\pi r}{T} = \frac{2\pi(10.0 \text{ m})}{0.75 \text{ s}} = 84 \text{ m/s}$$

5 $(2/3)\pi$ radians / s (2.1 radians/s)

1. Use the equation for the angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

2. Plug in the numbers:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{(3)(2\pi)}{9.0 \text{ s}} = \left(\frac{2}{3} \right) \pi \text{ radians/s} = 2.1 \text{ radians/s}$$

6 π radians / s (3.1 radians/s)

1. Use the equation for the angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

2. Plug in the numbers:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{(0.5)(2\pi)}{1.0 \text{ sec}} = \pi \text{ radians/s} = 3.1 \text{ radians/s}$$

7 120 minutes

1. Start with the equation for the angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

2. Solve for Δt :

$$\Delta t = \frac{\Delta\theta}{\omega}$$

3. Plug in the numbers:

$$\Delta t = \frac{\Delta\theta}{\omega} = \frac{2\pi}{8.7 \times 10^{-4} \text{ radians/s}} = 7,200 \text{ s}$$

That's about 120 minutes.

8 3.0 s

1. Start with the equation for the angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

2. Solve for Δt :

$$\Delta t = \frac{\Delta\theta}{\omega}$$

3. Plug in the numbers:

$$\Delta t = \frac{\Delta\theta}{\omega} = \frac{2\pi}{2.1 \text{ radians/s}} = 3.0 \text{ s}$$

9 30 radians/s²

1. Use the equation for angular acceleration:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

2. Plug in the numbers:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(5.0 \text{ radians/s} - 2.0 \text{ radians/s})}{10^{-1} \text{ s}} = 30 \text{ radians/s}^2$$

10 0.63 radians/s²

1. Use the equation for angular acceleration:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

2. Plug in the numbers:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(5.4 \text{ radians/s} - 3.5 \text{ radians/s})}{3.0 \text{ s}} = 0.63 \text{ radians/s}^2$$

11 6.0 radians/s

1. Use the equation for angular acceleration:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

2. Solve for $\Delta\omega$:

$$\Delta\omega = \alpha \Delta t$$

3. Plug in the numbers:

$$\Delta\omega = \alpha \Delta t = (10.0 \text{ radians/s}^2)(0.50 \text{ s}) = 5.0 \text{ radians/s}$$

4. Add $\Delta\omega$ to the original angular velocity, ω_o :

$$1.0 \text{ radian/s} + 5.0 \text{ radians/s} = 6.0 \text{ radians/s}$$

12 15 radians/s

1. Use the equation for angular acceleration:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

2. Solve for $\Delta\omega$:

$$\Delta\omega = \alpha \Delta t$$

3. Plug in the numbers:

$$\Delta\omega = \alpha \Delta t = (15 \text{ radians/s}^2)(1.0 \text{ s}) = 15 \text{ radians/s}$$

4. Add $\Delta\omega$ to the original angular velocity, ω_o :

$$0 \text{ radians/s} + 15 \text{ radians/s} = 15 \text{ radians/s}$$

13 68 radians

1. Use this equation:

$$\theta = \omega_o(t_f - t_o) + \frac{1}{2}\alpha(t_f - t_o)^2$$

2. Plug in the numbers:

$$\begin{aligned}\theta &= \omega_o(t_f - t_o) + \frac{1}{2}\alpha(t_f - t_o)^2 \\ &= (1.0 \text{ radian/s})(5.0 \text{ s}) + \frac{1}{2}(5.0 \text{ radians/s}^2)(5.0 \text{ s})^2 \\ &= 5.0 \text{ radians} + 62.5 \text{ radians} = 67.5 \text{ radians, which rounds to 68 radians with significant figures}\end{aligned}$$

14 360 radians

1. Use this equation:

$$\theta = \omega_o(t_f - t_o) + \frac{1}{2}\alpha(t_f - t_o)^2$$

2. Plug in the numbers:

$$\begin{aligned}\theta &= \omega_o(t_f - t_o) + \frac{1}{2}\alpha(t_f - t_o)^2 \\ &= (8.0 \text{ radians/s})(20.0 \text{ s}) + \frac{1}{2}(1.0 \text{ radian/s}^2)(20.0 \text{ s})^2 \\ &= 160 \text{ radians} + 200 \text{ radians} \\ &= 360 \text{ radians}\end{aligned}$$

15 1.0 radian/s²

1. Use this equation:

$$\theta = \omega_o(t_f - t_o) + \frac{1}{2}\alpha(t_f - t_o)^2$$

2. Solve for α , given that $t_o = 0$:

$$\alpha = \frac{2(\theta - \omega_o t_f)}{t_f^2}$$

3. Plug in the numbers:

$$\begin{aligned}\alpha &= \frac{2(\theta - \omega_o t_f)}{t_f^2} \\ &= \frac{2(60.0 \text{ radians} - [7.0 \text{ radians/s}][6.0 \text{ s}])}{(6.0 \text{ s})^2} \\ &= 1.0 \text{ radian/s}^2\end{aligned}$$

16 -1.6 radians/s²

1. Use this equation:

$$\theta = \omega_o(t_f - t_o) + \frac{1}{2}\alpha(t_f - t_o)^2$$

2. Solve for α , given that $t_o = 0$:

$$\alpha = \frac{2(\theta - \omega_o t_f)}{t_f^2}$$

3. Plug in the numbers:

$$\begin{aligned}\alpha &= \frac{2(\theta - \omega_o t_f)}{t_f^2} \\ &= \frac{2(40 \text{ radians} - [12.0 \text{ radians/s}][5.0 \text{ s}])}{(5.0 \text{ s})^2} \\ &= -1.6 \text{ radians/s}^2\end{aligned}$$

17 140 radians

1. Use this equation:

$$\omega_f^2 - \omega_o^2 = 2\alpha\theta$$

2. Solve for θ :

$$\theta = \frac{\omega_f^2 - \omega_o^2}{2\alpha}$$

3. Plug in the numbers:

$$\begin{aligned}\theta &= \frac{\omega_f^2 - \omega_o^2}{2\alpha} \\ &= \frac{(80 \text{ radians/s})^2 - (60 \text{ radians/s})^2}{2(10 \text{ radians/s}^2)} \\ &= 140 \text{ radians}\end{aligned}$$

18 11 radians

1. Use this equation:

$$\omega_f^2 - \omega_o^2 = 2\alpha\theta$$

2. Solve for θ :

$$\theta = \frac{\omega_f^2 - \omega_o^2}{2\alpha}$$

3. Plug in the numbers:

$$\begin{aligned}\theta &= \frac{\omega_f^2 - \omega_o^2}{2\alpha} \\ &= \frac{(24 \text{ radians/s}^2) - (12 \text{ radians/s})^2}{2(20 \text{ radians/s}^2)} \\ &= 11 \text{ radians}\end{aligned}$$

19 $1.3 \times 10^4 \text{ m/s}^2$

1. Use this equation:

$$a_c = \frac{v^2}{r}$$

2. Plug in the numbers:

$$a_c = \frac{v^2}{r} = \frac{(300 \text{ m/s})^2}{7.0 \text{ m}} = 1.3 \times 10^4 \text{ m/s}^2$$

20 360 m/s^2

1. Use this equation:

$$a_c = \frac{v^2}{r}$$

2. Convert the 60 miles/hour speed to m/s:

$$\frac{60 \text{ miles}}{\text{hour}} \times \frac{1.609 \text{ km}}{\text{mile}} \times \frac{1,000 \text{ m}}{\text{km}} \times \frac{1 \text{ hour}}{3,600 \text{ s}} = 26.8 \text{ m/s}$$

3. Plug in the numbers:

$$a_c = \frac{v^2}{r} = \frac{(26.8 \text{ m/s})^2}{2.0 \text{ m}} = 360 \text{ m/s}^2$$

21 13 N

1. Use the equation for a centripetal force:

$$F_c = m\omega^2 r$$

2. Plug in the numbers:

$$\begin{aligned} F_c &= m\omega^2 r \\ &= (0.10 \text{ kg})(8.0 \text{ radians/s})^2 (2.0 \text{ m}) \\ &= 13 \text{ N} \end{aligned}$$

22 360 N

1. Use this equation for a centripetal force:

$$F_c = m\omega^2 r$$

2. Plug in the numbers:

$$\begin{aligned} F_c &= m\omega^2 r \\ &= (1.0 \text{ kg})(6.0 \text{ radians/s})^2 (10 \text{ m}) \\ &= 360 \text{ N} \end{aligned}$$

3

**Being Energetic:
Work**

IN THIS PART . . .

Work through problems about kinetic energy, potential energy, and power.

Predict what will happen when objects collide.

Figure out what happens when a force is applied to the edge of a rotating object.

Get going with inertia, torque, and rotational kinetic energy.

Solve problems of simple harmonic motion and springs.

IN THIS CHAPTER

- » Understanding work
- » Working with net force
- » Calculating kinetic energy
- » Handling potential energy
- » Relating kinetic energy to work

Chapter 7

Working the Physics Way

This chapter covers all kinds of problems having to do with work in physics terms. Work has a specific definition in physics: It's force times the distance over which that force acts. For example, you may be holding up a refrigerator and thinking that you're doing a lot of work, but if that refrigerator is stationary, it's not considered work in physics. (On the other hand, lots of things are happening in your body at the molecular level to let you hold that refrigerator up, and many of those things result in work — but that's another story, a biophysics one.)

In addition to work, this chapter covers kinetic energy — that is, the ability to do work that comes from motion. Need an example? If you're at the bottom of a hill and trying to stop a speeding car, you get an instant introduction to kinetic energy. Also covered in this chapter is potential energy, which is the ability to do work stored in an object. For example, a car at the bottom of a hill has less potential energy than one at the top of a hill because the one at the top of the hill can travel down, converting its potential energy into kinetic energy.

Finally, this chapter considers problems of power, or how much work you do in so much time. That's measured in watts, which you're surely familiar with thanks to light bulbs.

A Different Kind of Work

In physics, *physical work* is defined as the applied force multiplied by the distance over which it was applied. So if you're pushing a refrigerator 2.0 m across the floor, and you need to apply 900 N, you've done this much work:

$$(900)(2.0) = 1,800$$

1,800 what? Work in the MKS system is measured in joules, J, so that's 1,800 J. You should almost always see work expressed in joules in physics problems. The CGS unit for work is the (oddly named) erg, but it's rare that you'll see that used.

So applying a force of 1.0 N over a distance of 1.0 m means that you've done 1 joule of work. $1.0 \text{ J} = 1.0 \times 10^7$ ergs because ergs are dyne-centimeters, and there are 10^5 dynes per Newton and 10^2 centimeters in a meter. The FPS system of work uses the foot-pound as the unit for work.

But work isn't just the total force applied multiplied by the distance traveled; the only component of a force that counts is the one applied along the direction of travel. For example, look at the mass being pulled in Figure 7-1.

FIGURE 7-1:
Dragging a
mass.



© John Wiley & Sons, Inc.

The force applied along the direction of travel is what counts, so you want the component of F that is horizontal (assuming you're dragging the mass and not picking it up!). That horizontal component is $F \cos \theta$, so the work you do, W , is this:

$$W = Fs \cos \theta$$

Here, F is the force applied, s is the displacement, and θ is the angle between them.



Q. Suppose that you're pulling the mass in Figure 7-1 by applying 200 N and the angle θ is 40.0° . How much work do you do dragging the mass over 50.0 m?

EXAMPLE

A. The correct answer is 7,660 J.

1. Use the equation $W = Fs \cos \theta$
2. Plug in the numbers:

$$W = Fs \cos \theta = (200 \text{ N})(50.0 \text{ m})(0.766) = 7,660 \text{ J}$$

1 You're pulling a chest of drawers, applying a force of 60.0 N at an angle of 60.0° . How much work do you do pulling it over 10.0 m?

2 You're dragging a sled on a rope at 30° , applying a force of 20 N. How much work do you do over 1.0 km?

3 You're pushing a box of dishes across the kitchen floor, using 100.0 J to move it 10.0 m. If you apply the force at 60.0° , what is the force you used?

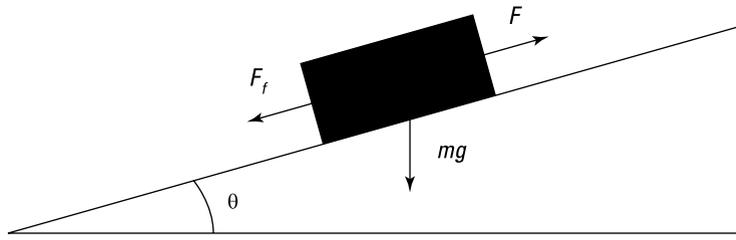
4 You're pushing an out-of-gas car down the road, applying a force of 800.0 N. How much work have you done in moving the car 10.0 m?

Dealing with the Net Force

There's often more than one force involved when you're dragging a mass over a distance. Just think of the forces of friction and gravity.

For example, take a look at Figure 7-2, where a couch is being dragged up an incline. If you're applying force F , how much work is done as the couch is dragged up the incline?

FIGURE 7-2: Dragging a mass up an incline.



© John Wiley & Sons, Inc.

Look at the forces involved in this scenario: There's the force you apply (your pull), the force due to friction that resists your pull, and the force due to gravity that also resists your pull. While all these forces do work on the couch as it's being pulled up the slope, the net work done on the couch is the product of the component of the *net* force multiplied by the distance traveled.

In other words, you may do 100 J of work on the couch; friction, which is opposing you, may do -50 J (the work is negative because the force here is opposite to the direction traveled); and the force of gravity may do -30 J. That means that a net work of 20 J was done on the couch by all the forces ($100 - 50 - 30 = 20$ J).



EXAMPLE

Q. Suppose that the couch you're dragging in Figure 7-2 has a mass of 75.0 kg, and the angle of the ramp is 24.0° . If the coefficient of kinetic friction is 0.170 and you're pulling the couch 2.00 m with a force of 800.0 N, how much work is being done by the net force on the couch?

A. The correct answer is 774 J.

1. Identify the forces on the couch. You're pulling with a force F in the direction of the ramp, so the component of your force in the direction of travel is exactly 800 N.
2. Look at the component of the force due to gravity downhill along the direction of the incline. The figure tells you that this force is $F_g = mg \sin \theta$. (You can verify this by noting that F_g goes to mg as θ goes to 90° and to 0 as θ goes to 0° .)
3. Calculate the force due to friction. That's $F_f = \mu_k F_n$, where F_n is the normal force. The normal force in this scenario is $mg \cos \theta$, so that makes the force due to friction

$$F_f = \mu_k F_n = \mu_k mg \cos \theta$$

4. Calculate the net force (upward along the incline):

$$\Sigma F = F - mg \sin \theta - \mu_k mg \cos \theta$$

Note that the forces due to friction and gravity are negative because they oppose the direction of travel.

5. Because all forces act along the direction of travel, the net work, W , is equal to

$$W = \Sigma Fs = (F - mg \sin \theta - \mu_k mg \cos \theta)s.$$

6. Plug in the numbers:

$$\begin{aligned} W &= \Sigma Fs = (F - mg \sin \theta - \mu_k mg \cos \theta)s \\ &= (800.0 \text{ N} - [75.0 \text{ kg}][9.8 \text{ m/s}^2][0.41] - [0.17][9.8 \text{ m/s}^2][75.0 \text{ kg}][0.91])(2.00 \text{ m}) \\ &= (800.0 \text{ N} - 299 \text{ N} - 114 \text{ N})(2.00 \text{ m}) \\ &= 774 \text{ J} \end{aligned}$$

5 You're pulling a chest of drawers, applying a force of 600.0 N parallel to the slope. The angle of the slope is 40.0° . The chest has a mass of 40.0 kg, and there's a coefficient of kinetic friction of 0.12. How much net work do you do in pulling the chest of drawers 10.0 m up the inclined plane?

6 You're applying a force of 800.0 N to the ground as you go up a ski slope on your new skis; this force is applied parallel to the slope (downhill). Thus, the ground is necessarily pushing you with an equal force of 800.0 N in the opposite direction (uphill). If the slope is 22.0° , the coefficient of kinetic friction is 0.050, and you have a mass of 80.0 kg, how much work does the ground do pushing you up that slope 500.0 m? How much net work is done?

Getting Energetic: Kinetic Energy

When you have objects in motion, you have kinetic energy. When, for example, you slide an object on a frictionless surface, the work you do goes into the object's kinetic energy:

$$W = KE_f - KE_i$$

If you have an object with mass m moving at speed v , its kinetic energy is

$$KE = \frac{1}{2}mv^2$$

That's the energy of motion.



Q. Say that you push a spaceship, mass 1,000.0 kg, by applying a force of 1.00×10^4 N for 1.00 m. How fast does the spaceship end up traveling?

EXAMPLE

A. The correct answer is 4.47 m/s.

1. Find the total work done:

$$(1.00 \times 10^4 \text{ N})(1.00 \text{ m}) = 1.00 \times 10^4 \text{ J}$$

2. The work done on the spaceship goes into its kinetic energy, and its initial kinetic energy is 0, so its final kinetic energy is 1.00×10^4 J.

3. Use the equation for kinetic energy and solve for v :

$$\begin{aligned} KE &= \frac{1}{2}mv^2 \\ 1.00 \times 10^4 \text{ J} &= \frac{1}{2}(1,000.0 \text{ kg})v^2 \\ v^2 &= \frac{(1.00 \times 10^4 \text{ J})}{500.0 \text{ kg}} \\ v^2 &= 20.0 \text{ m}^2/\text{s}^2 \\ v &= \sqrt{20.0} \text{ m/s} = 4.47 \text{ m/s} \end{aligned}$$

The final speed is $20.0^{1/2}$ m/s = 4.47 m/s.

7 You're applying 600.0 N of force to a car with a mass of 1,000.0 kg. You're traveling a distance of 100.0 m on a frictionless, icy road. What is the car's final speed?

8 You're ice skating and traveling at 30.0 m/s. If your mass is 65 kg, what is your kinetic energy?

- 9 You're traveling in a car at 88 m/s. If you have a mass of 80.0 kg and the car has a mass of 1,200.0 kg, what is the total kinetic energy of you and the car combined?

- 10 You're applying 3,000.0 N of force to a hockey puck with a mass of 0.10 kg. It travels over a distance of 0.10 m on a frictionless ice rink. What is the puck's final speed?

Getting Kinetic Energy from Work

Here's more on kinetic energy.



EXAMPLE

- Q.** In Figure 7-3, a 1,000.0-kg safe full of gold bars is sliding down a 3.0-m ramp that meets the horizontal at an angle of 23° . The coefficient of kinetic friction is 0.15. What will the refrigerator's speed be when it reaches the bottom of the ramp?

- A.** The correct answer is 3.8 m/s.

1. Determine the net force acting on the refrigerator. The component of the refrigerator's weight acting along the ramp is

$$F = mg \sin \theta$$

2. You know that the normal force is

$$F_n = mg \cos \theta$$

That means that the kinetic force of friction is

$$F_f = \mu_k F_n = \mu_k mg \cos \theta$$

3. The net force accelerating the safe down the ramp, F_{net} , is

$$F_{net} = mg \sin \theta - F_f = mg \sin \theta - \mu_k mg \cos \theta$$

4. Plug in the numbers:

$$\begin{aligned}F_{net} &= (1,000 \text{ kg})(9.8 \text{ m/s}^2)(\sin 23^\circ) - (0.15)(1,000 \text{ kg})(9.8 \text{ m/s}^2)(\cos 23^\circ) \\ &= 3,800 \text{ N} - 1,350 \text{ N} \\ &= 2,450 \text{ N}\end{aligned}$$

5. The net force, 2,450 N, acts over the entire 3.0-m ramp, so the work done by this force is

$$W = F_{net}s = (2,450 \text{ N})(3.0 \text{ m}) = 7,400 \text{ J}$$

6. 7,400 J of work is done on the safe by the net force, and that work goes into the safe's kinetic energy. You find the safe's kinetic energy by using this equation:

$$W = F_{net}s = 7,400 \text{ J} = \frac{1}{2}mv^2$$

7. Solve for v^2 :

$$v^2 = 14.8 \text{ m}^2/\text{s}^2, \text{ so } v = 3.8 \text{ m/s}$$

11 A 40.0-kg box of books is sliding down a 4.0-m ramp of 27° . If the coefficient of kinetic friction is 0.17, what is the box's speed at the bottom of the ramp?

12 An 80.0-kg person trips and slides down a 20.0-m toboggan run of 27° . If the coefficient of kinetic friction is 0.10, what is the person's speed at the bottom of the run?

- 13 A parked car with its wheels locked starts sliding down an icy street with an angle of 28° . If the coefficient of kinetic friction is 0.10, the street is 40.0 m to the bottom, and the car has a mass of 1,000.0 kg, what is the car's speed at the bottom of the street?

- 14 You're going down a 100.0-m ski jump at an angle of 40.0° . Your mass (including skis!) is 90.0 kg, and the coefficient of kinetic friction is 0.050. What is your speed at the bottom of the jump?

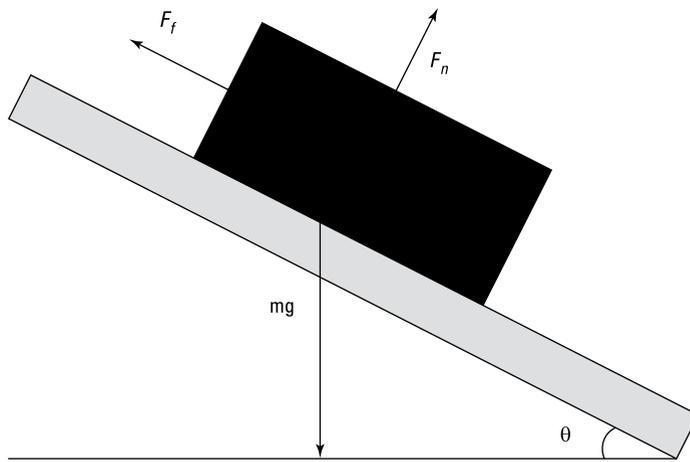


FIGURE 7-3:
The safe slides down.

© John Wiley & Sons, Inc.

Storing Your Energy: Potential Energy

Objects can have energy at rest simply by having a force act on them. For example, an object at the end of a stretched spring has energy because when you let the object go and it accelerates because of the spring, it can convert that stored energy into kinetic energy.

The energy that an object has by virtue of a force acting on it is called *potential energy*. For example, an object at height h in the gravitational field at Earth's surface has this potential energy:

$$PE = mgh$$

For example, say you have a basketball at height h . When it drops, its potential energy gets converted into kinetic energy. To figure out how fast the basketball (or any object) is going when it hits the ground, you use this equation:

$$PE = KE = mgh = \frac{1}{2}mv^2$$



EXAMPLE

Q. During a basketball game, a 1.0-kg ball gets thrown vertically in the air. It's momentarily stationary at a height of 5.0 m above the floor and then falls back down. What is the ball's speed when it hits the floor?

A. The correct answer is 9.9 m/s.

1. Find out how much potential energy the ball has when it starts to fall. Use this equation for potential energy:

$$PE = mgh$$

2. This potential energy will be converted into kinetic energy, so you expand the equation like so:

$$PE = mgh = KE = \frac{1}{2}mv^2$$

3. Solve for v^2 :

$$v^2 = 2gh$$

4. Solve for v :

$$v = \sqrt{2gh}$$

5. Plug in the numbers:

$$\begin{aligned} v &= \sqrt{2gh} \\ &= \sqrt{2(9.8 \text{ m/s}^2)(5.0 \text{ m})} = 9.9 \text{ m/s} \end{aligned}$$

15 A 40-kg box of books falls off a shelf that's 4.0 m above the ground. How fast is the box traveling when it hits the ground?

16 You jump out of an airplane at 2,000 ft and fall 1,000 ft before opening your parachute. What is your speed in ft/s (neglecting air resistance) when you open your chute?

17 The flagpole on top of a 300.0-m skyscraper falls off. How fast is it falling in m/s when it strikes the ground? (Assume that you can neglect air resistance.)

18 If you're in an airplane at 30,000 ft, what is your potential energy if you have a mass of 70.0 kg?

Powering It Up

When it comes to work in physics, you're sure to see problems involving *power*, which is the amount of work being done in a certain amount of time. Here's the equation for power, P :

$$P = \frac{W}{t}$$

W equals force along the direction of travel times distance, so you could write the equation for power this way:

$$P = \frac{W}{t} = \frac{Fs\cos\theta}{t}$$

where θ is the angle between the force and the direction of travel. On the other hand, the object's speed, v , is just s/t (displacement over time), so the equation breaks down further to:

$$P = \frac{W}{t} = \frac{Fs\cos\theta}{t} = Fv\cos\theta$$

In the special case where the force acts along the direction of travel, you have the simplified formula:

$$P = \frac{W}{t} = \frac{Fs}{t} = Fv$$

So in this simplest case, power equals force times speed. You use this equation when you need to apply a force to keep something moving at constant speed.



EXAMPLE

Q. You're riding a toboggan down an icy run to a frozen lake, and you accelerate the 80.0-kg combination of you and the toboggan from 1.0 m/s to 2.0 m/s in 2.0 s. How much power does that require?

A. The correct answer is 60 watts.

1. Assuming that there's no friction on the ice, you use this equation for the total work:

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

2. Plug in the numbers:

$$\begin{aligned} W &= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \\ &= \frac{1}{2}(80.0 \text{ kg})(2.0 \text{ m/s})^2 - \frac{1}{2}(80.0 \text{ kg})(1.0 \text{ m/s})^2 \\ &= 120 \text{ J} \end{aligned}$$

3. Because this work is done in 2 s, the power involved is

$$P = \frac{120 \text{ J}}{2.0 \text{ s}} = 60 \text{ W}$$

19 A 1,000-kg car accelerates from 88 m/s to 100 m/s in 30 s. How much power does that require?

20 A 60.0-kg person is running and accelerates from 5.0 m/s to 7.0 m/s in 2.0 s. How much power does that require?

21 A 120-kg linebacker accelerates from 5.0 m/s to 10.0 m/s in 1.0 s. How much power does that require?

22 You're driving a snowmobile that accelerates from 10 m/s to 20 m/s over a time interval of 10.0 s. If you and the snowmobile together have a mass of 500 kg, how much power is used?

Answers to Problems about Work

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 **300 J**

1. Use the equation $W = Fs \cos \theta$.
2. Plug in the numbers:

$$W = Fs \cos \theta = (60.0 \text{ N})(10.0 \text{ m})(0.5) = 300 \text{ J}$$

2 **$1.7 \times 10^4 \text{ J}$**

1. Use the equation $W = Fs \cos \theta$.
2. Plug in the numbers:

$$W = Fs \cos \theta = (20 \text{ N})(1,000 \text{ m})(0.866) = 1.7 \times 10^4 \text{ J}$$

3 **20 N**

1. Use the equation $W = Fs \cos \theta$.
2. Solve for force:

$$F = \frac{W}{s \cos \theta}$$

3. Plug in the numbers:

$$F = \frac{W}{s \cos \theta} = \frac{100.0 \text{ J}}{(10.0 \text{ m})(0.5)} = 20 \text{ N}$$

4 **8,000 J**

1. Use the equation $W = Fs \cos \theta$.
2. Plug in the numbers:

$$W = Fs \cos \theta = (800.0 \text{ N})(10.0 \text{ m})(1) = 8,000 \text{ J}$$

5 **3,120 J**

1. Find the forces on the chest of drawers. You're pulling with a force F in the direction of the ramp, so the component of your force in the direction of travel is 600.0 N.
2. Find the force due to gravity along the direction of the ramp. Keep in mind that this force will be going down the ramp.

$$F_g = mg \sin \theta$$

3. Find the force due to friction. That's $F_f = \mu_k F_n$, where F_n is the normal force. The normal force here is $mg \cos \theta$, so that makes the force due to friction

$$F_f = \mu_k F_n = \mu_k mg \cos \theta$$

4. Calculate the net force:

$$\Sigma F = F - mg \sin \theta - \mu_k mg \cos \theta$$

Note that the force due to friction and gravity are negative because they oppose the direction of travel.

5. The net work done is the total force along the direction of travel times the distance traveled. Because all forces act along the direction of travel, the net work, W , is

$$W = \Sigma Fs = (F - mg \sin \theta - \mu_k mg \cos \theta)s$$

6. Plug in the numbers:

$$\begin{aligned} W &= \Sigma Fs = (F - mg \sin \theta - \mu_k mg \cos \theta)s \\ &= (600.0 \text{ N} - [40.0 \text{ kg}][9.8 \text{ m/s}^2][0.64] - [0.12][9.8 \text{ m/s}^2][40.0 \text{ kg}][0.77])(10.0 \text{ m}) \\ &= (600.0 \text{ N} - 252 \text{ N} - 36 \text{ N})(10.0 \text{ m}) = 3,120 \text{ J} \end{aligned}$$

6 The ground does 4.00×10^5 J of work. The net work is 2.35×10^5 J.

1. The ground is pushing you up the slope with a force of 800.0 N; this force goes along the direction of the slope.
2. The force due to gravity parallel to the slope is $F_g = mg \sin \theta$. This force is going down the slope.
3. The force due to friction is $F_f = \mu_k F_n$, where F_n is the normal force. The normal force here is $mg \cos \theta$, so that makes the force due to friction:

$$F_f = \mu_k F_n = \mu_k mg \cos \theta$$

4. Calculate the net force:

$$\Sigma F = F - mg \sin \theta - \mu_k mg \cos \theta$$

5. The net work done is the total force along the direction of travel times the distance traveled. Because all forces act along the direction of travel, the net work, W_{net} , is

$$W_{net} = \Sigma Fs = (F - mg \sin \theta - \mu_k mg \cos \theta)s$$

6. Plug in the numbers to find the net work:

$$\begin{aligned} W_{net} &= \Sigma Fs = (F - mg \sin \theta - \mu_k mg \cos \theta)s \\ &= (800.0 \text{ N} - [80.0 \text{ kg}][9.8 \text{ m/s}^2][0.37] - [0.050][9.8 \text{ m/s}^2][80.0 \text{ kg}][0.93])(500.0 \text{ m}) \\ &= (800.0 \text{ N} - 294 \text{ N} - 36 \text{ N})(500.0 \text{ m}) \\ &= 2.35 \times 10^5 \text{ J} \end{aligned}$$

7. The work done by the ground pushing on you is $(800.0 \text{ N})(500.0 \text{ m}) = 4.000 \times 10^5$ J.

7 11 m/s

1. Find the total work done:

$$(600 \text{ N})(100 \text{ m}) = 6.0 \times 10^4 \text{ J}$$

- The work done on the car goes into its kinetic energy, so its final kinetic energy is 6.0×10^4 J.
- Use the equation for kinetic energy and solve for v :

$$KE = \frac{1}{2}mv^2$$

$$v^2 = \frac{2}{m}(KE)$$

- Plug in the numbers:

$$v^2 = \frac{2(6 \times 10^4 \text{ J})}{1,000 \text{ kg}} = 120$$

The final speed = $120^{1/2}$ m/s = 11 m/s.

8 2.9×10^4 J

- The equation for kinetic energy is

$$KE = \frac{1}{2}mv^2$$

- Plug in the numbers:

$$\begin{aligned} \frac{1}{2}mv^2 &= \frac{1}{2}(65 \text{ kg})(30.0 \text{ m/s})^2 \\ &= 2.9 \times 10^4 \text{ J} \end{aligned}$$

9 4.9×10^6 J

- The equation for kinetic energy is

$$KE = \frac{1}{2}mv^2$$

- Plug in the numbers:

$$\begin{aligned} \frac{1}{2}(m + M)v^2 &= \frac{1}{2}(80.0 \text{ kg} + 1,200.0 \text{ kg})(88 \text{ m/s})^2 \\ &= 4.9 \times 10^6 \text{ J} \end{aligned}$$

10 77 m/s

- Find the total work done:

$$(3,000 \text{ N})(0.10 \text{ m}) = 300 \text{ J}$$

- The work done on the puck goes into its kinetic energy, so its final kinetic energy is 300 J.
- Use the equation for kinetic energy and solve for v :

$$KE = \frac{1}{2}mv^2$$

$$v^2 = \frac{2}{m}(KE)$$

4. Plug in the numbers:

$$v^2 = \frac{2(300 \text{ J})}{0.10 \text{ kg}} = 6,000$$

The final speed = $6,000^{1/2}$ m/s = 77 m/s.

11 **4.9 m/s**

1. To find the net force on the box, start with the component of the box's weight acting along the ramp:

$$mg \sin \theta$$

2. The normal force on the box is

$$F_n = mg \cos \theta$$

which means the kinetic force of friction is

$$F_f = \mu_k F_n = \mu_k mg \cos \theta$$

3. The net force accelerating the box down the ramp, F_{net} , is

$$F_{net} = mg \sin \theta - F_f = mg \sin \theta - \mu_k mg \cos \theta$$

4. Plug in the numbers:

$$\begin{aligned} F_{net} &= (40.0 \text{ kg})(9.8 \text{ m/s}^2)(\sin 27^\circ) - (0.17)(40.0 \text{ kg})(9.8 \text{ m/s}^2)(\cos 27^\circ) \\ &= 180 \text{ N} - 60 \text{ N} \\ &= 120 \text{ N} \end{aligned}$$

5. The net force is 120 N. This net force acts over the 4.0-m ramp, so the work done by this force is

$$W = F_{net} S = (120 \text{ N})(4.0 \text{ m}) = 480 \text{ J}$$

6. 480 J of work is done on the box, and that work goes into the box's kinetic energy. Find the box's kinetic energy:

$$W = F_{net} s = 480 \text{ J} = \frac{1}{2} mv^2$$

7. Solve:

$$\begin{aligned} 480 \text{ J} &= \frac{1}{2} mv^2 \\ 480 \text{ J} &= \frac{1}{2} (40.0 \text{ kg})v^2 \\ v^2 &= 24 \text{ m}^2 / \text{s}^2 \\ v &= 4.9 \text{ m/s} \end{aligned}$$

12 **11.8 m/s**

1. Find the net force on the sliding person. The component of the person's weight acting along the run is

$$F = mg \sin \theta$$

2. The normal force on the person is

$$F_n = mg \cos \theta$$

which means that the kinetic force of friction is

$$F_f = \mu_k F_n = \mu_k mg \cos \theta$$

3. The net force accelerating the person down the ramp, F_{net} , is

$$F_{net} = mg \sin \theta - F_f = mg \sin \theta - \mu_k mg \cos \theta$$

4. Plug in the numbers:

$$\begin{aligned} F_{net} &= (80.0 \text{ kg})(9.8 \text{ m/s}^2)(\sin 27^\circ) - (0.10)(80.0 \text{ kg})(9.8 \text{ m/s}^2)(\cos 27^\circ) \\ &= 350 \text{ N} - 70 \text{ N} \\ &= 280 \text{ N} \end{aligned}$$

5. The net force acts over the 20.0-m toboggan ramp, so the work done by this force is

$$W = F_{net}s = (280 \text{ N})(20.0 \text{ m}) = 5,600 \text{ J}$$

6. 5,600 J of work is done on the person, and that work goes into the person's kinetic energy. Find the person's kinetic energy:

$$W = F_{net}s = 5,600 \text{ J} = \frac{1}{2}mv^2$$

7. Solve:

$$\begin{aligned} 5,600 \text{ J} &= \frac{1}{2}mv^2 \\ 5,600 \text{ J} &= \frac{1}{2}(80.0 \text{ kg})v^2 \\ v^2 &= 140 \text{ m}^2/\text{s}^2 \\ v &= 11.8 \text{ m/s} \end{aligned}$$

13 **17 m/s**

1. Find the net force on the sliding car. The component of the car's weight acting along the street is

$$F = mg \sin \theta$$

2. The normal force on the person is

$$F_n = mg \cos \theta$$

which means that the kinetic force of friction is

$$F_f = \mu_k F_n = \mu_k mg \cos \theta$$

3. The net force accelerating the car down the street, F_{net} , is

$$F_{net} = mg \sin \theta - F_f = mg \sin \theta - \mu_k mg \cos \theta$$

4. Plug in the numbers:

$$\begin{aligned} F_{net} &= (1,000.0 \text{ kg})(9.8 \text{ m/s}^2)(\sin 28^\circ) - (0.10)(1,000.0 \text{ kg})(9.8 \text{ m/s}^2)(\cos 28^\circ) \\ &= 4,600 \text{ N} - 865 \text{ N} \\ &= 3,700 \text{ N} \end{aligned}$$

5. This net force acts over the 40-m street, so the work done by this force is

$$W = F_{net}s = (3,700 \text{ N})(40.0 \text{ m}) = 1.5 \times 10^5 \text{ J}$$

6. That work goes into the car's kinetic energy:

$$W = F_{net}s = 1.5 \times 10^5 \text{ J} = \frac{1}{2}mv^2$$

7. Solve:

$$\begin{aligned} 1.5 \times 10^5 \text{ J} &= \frac{1}{2}mv^2 \\ 1.5 \times 10^5 \text{ J} &= \frac{1}{2}(1,000.0 \text{ kg})v^2 \\ v^2 &= 300 \text{ m}^2/\text{s}^2 \\ v &= 17 \text{ m/s} \end{aligned}$$

14 **35 m/s**

1. Find the net force on yourself to start. The component of your weight acting along the ski jump is

$$F = mg \sin \theta$$

2. The normal force on yourself is

$$F_n = mg \cos \theta$$

which means the kinetic force of friction is

$$F_f = \mu_k F_n = \mu_k mg \cos \theta$$

3. The net force accelerating you down the ski jump, F_{net} , is

$$F_{net} = mg \sin \theta - F_f = mg \sin \theta - \mu_k mg \cos \theta$$

4. Plug in the numbers:

$$\begin{aligned} F_{net} &= (90.0 \text{ kg})(9.8 \text{ m/s}^2)(\sin 40^\circ) - (0.05)(90.0 \text{ kg})(9.8 \text{ m/s}^2)(\cos 40^\circ) \\ &= 570 \text{ N} - 34 \text{ N} \\ &= 540 \text{ N} \end{aligned}$$

5. This net force acts over the 100-m jump, so the work done by this force is

$$W = F_{net}s = (540 \text{ N})(100.0 \text{ m}) = 5.4 \times 10^4 \text{ J}$$

6. That work goes into your kinetic energy:

$$W = F_{net}s = 5.4 \times 10^4 \text{ J} = \frac{1}{2}mv^2$$

7. Solve:

$$5.4 \times 10^4 \text{ J} = \frac{1}{2}mv^2$$

$$5.4 \times 10^4 \text{ J} = \frac{1}{2}(90.0 \text{ kg})v^2$$

$$v^2 = 1,200 \text{ m}^2/\text{s}^2$$

$$v = 35 \text{ m/s}$$

15 8.9 m/s

1. Figure out how much potential energy the box has when it starts to fall. That potential energy is

$$PE = mgh$$

2. This potential energy will be converted into kinetic energy, so you have

$$PE = KE = mgh = \frac{1}{2}mv^2$$

3. Solve for v^2 :

$$v^2 = 2gh$$

4. Solve for v :

$$v = \sqrt{2gh} = \sqrt{2(9.8 \text{ m/s}^2)(4.0 \text{ m})} = 8.9 \text{ m/s}$$

16 254 ft/s

1. Figure out how much potential energy you have when you start to fall. That potential energy is

$$PE = mgh_o$$

2. Find out how much potential energy is converted into kinetic energy when you open your parachute. The change in potential energy, ΔPE , is

$$\Delta PE = mgh_o - mgh_f$$

3. This potential energy will be converted into kinetic energy, so you have

$$\Delta PE = mgh_o - mgh_f = KE = \frac{1}{2}mv^2$$

4. Note that in the FPI system of units, $g = 32.2 \text{ ft/s}^2$. Solve:

$$v^2 = 2g(h_o - h_f)$$

$$v^2 = 2(32.2 \text{ ft/s}^2)(2,000 \text{ ft} - 1,000 \text{ ft})$$

$$v^2 = 64,400 \text{ ft}^2 / \text{s}^2$$

$$v = 254 \text{ ft/s}$$

17 **77 m/s**

1. Figure out how much potential energy the pole has when it starts to fall. That potential energy is

$$PE = mgh$$

2. This potential energy will be converted into kinetic energy, so you have

$$PE = KE = mgh = \frac{1}{2}mv^2$$

3. Solve for v^2 :

$$v^2 = 2gh$$

4. Plug in the numbers:

$$v = \sqrt{2gh} = \sqrt{2(9.8 \text{ m/s}^2)(300.0 \text{ m})} = 77 \text{ m/s}$$

18 **$6.3 \times 10^6 \text{ J}$**

1. The equation for potential energy is

$$PE = mgh$$

2. Convert 30,000 ft into meters using the fact that 1 ft = 0.305 m:

$$30,000 \text{ ft} \times \frac{0.305 \text{ m}}{\text{ft}} = 9,150 \text{ m}$$

3. Plug in the numbers:

$$PE = mgh = (70.0 \text{ kg})(9.8 \text{ m/s}^2)(9,150 \text{ m}) = 6.3 \times 10^6 \text{ J}$$

19 **$3.8 \times 10^4 \text{ W}$**

1. The equation for power is

$$P = \frac{W}{t}$$

2. The amount of work done is the difference in kinetic energy:

$$W = \frac{m}{2}(v_2^2 - v_1^2)$$

3. Therefore, the power is

$$P = \frac{W}{t} = \frac{m}{2t}(v_2^2 - v_1^2)$$

4. Plug in the numbers:

$$\begin{aligned} P &= \frac{W}{t} = \frac{m}{2t}(v_2^2 - v_1^2) \\ &= \frac{1,000.0 \text{ kg}}{2(30 \text{ s})}([100 \text{ m/s}]^2 - [88 \text{ m/s}]^2) \\ &= 3.8 \times 10^4 \text{ W} \end{aligned}$$

20 **360 W**

1. The equation for power is

$$P = \frac{W}{t}$$

2. The amount of work done is the difference in kinetic energy:

$$W = \frac{m}{2}(v_2^2 - v_1^2)$$

3. Therefore, the power is

$$P = \frac{W}{t} = \frac{m}{2t}(v_2^2 - v_1^2)$$

4. Plug in the numbers:

$$\begin{aligned} P &= \frac{W}{t} = \frac{m}{2t}(v_2^2 - v_1^2) \\ &= \frac{60.0 \text{ kg}}{2(2.0 \text{ s})}([7.0 \text{ m/s}]^2 - [5.0 \text{ m/s}]^2) \\ &= 360 \text{ W} \end{aligned}$$

21 **4,500 W**

1. The equation for power is

$$P = \frac{W}{t}$$

2. The amount of work done is the difference in kinetic energy:

$$W = \frac{m}{2}(v_2^2 - v_1^2)$$

3. Therefore, the power is

$$P = \frac{W}{t} = \frac{m}{2t}(v_2^2 - v_1^2)$$

4. Plug in the numbers:

$$\begin{aligned} P &= \frac{W}{t} = \frac{m}{2t}(v_2^2 - v_1^2) \\ &= \frac{120 \text{ kg}}{2(1.0 \text{ s})}([10.0 \text{ m/s}]^2 - [5.0 \text{ m/s}]^2) \\ &= 4,500 \text{ W} \end{aligned}$$

22 $7.5 \times 10^3 \text{ W}$

1. The equation for power is

$$P = \frac{W}{t}$$

2. The amount of work done is the difference in kinetic energy:

$$W = \frac{m}{2}(v_2^2 - v_1^2)$$

3. Therefore, the power is

$$P = \frac{W}{t} = \frac{m}{2t}(v_2^2 - v_1^2)$$

4. Plug in the numbers:

$$\begin{aligned} P &= \frac{W}{t} = \frac{m}{2t}(v_2^2 - v_1^2) \\ &= \frac{500 \text{ kg}}{2(10.0 \text{ s})}([20 \text{ m/s}]^2 - [10 \text{ m/s}]^2) \\ &= 7.5 \times 10^3 \text{ W} \end{aligned}$$

IN THIS CHAPTER

- » Getting a feel for impulse
- » Gaining momentum
- » Conserving momentum
- » Conserving kinetic energy
- » Handling two-dimensional collisions

Chapter 8

Getting Things to Move: Momentum and Kinetic Energy

Things collide all the time, and when they do, physics gets involved. Knowing about the conservation of momentum and kinetic energy lets you handle this kind of problem — when things hit each other, you can predict what's going to happen.

The conservation of momentum and kinetic energy uses two of the strongest tools that physics has in the physical world, and this chapter helps you become a pro at working with these tools.

Acting on Impulse

When you apply a force for a certain amount of time, you create an *impulse*. In fact, that's the definition of impulse — impulse equals the force applied multiplied by the time it was applied. Here's the equation:

$$\text{Impulse} = Ft$$

Note that this is a vector equation because the force has a direction; therefore, the impulse does as well (for a review of vector equations, check out Chapter 3). Impulse can be an important quantity when you're solving physics problems because you can relate impulse to momentum (which will be discussed in the next section), and working with momentum is how you solve collision problems in physics.

Here's an example of impulse in action: You're playing pool, and you strike a pool ball with the cue. The cue may be in contact with the ball for only a millisecond, but there's an observable result — the ball is now in motion. That is a result of impulse.

What are the units of impulse? You have force multiplied by time, so the unit is the Newton-second.



EXAMPLE

Q. Suppose that you're playing pool and hit a pool ball for 10.0 milliseconds (a millisecond is 1/1,000 of a second) with a force of 20.0 N. What impulse did you impart to the pool ball?

A. The correct answer is 0.200 N-s, in the direction of the force.

1. Use the equation **Impulse** = Ft .

2. Plug in the numbers:

$$\mathbf{Impulse} = Ft = (20.0 \text{ N})(1.0 \times 10^{-2} \text{ s}) = 0.200 \text{ N-s, in the direction of the force.}$$

1 You're disgusted with your computer and give it a whack. If your hand is in contact with the computer for 100.0 milliseconds with a force of 100.0 N, what impulse do you impart to the computer?

2 You're standing under the eaves of your house when a huge icicle breaks off and hits you, imparting a force of 300.0 N for 0.10 seconds. What was the impulse?

Getting Some Momentum

Momentum is the most important quantity when it comes to handling collisions in physics. *Momentum* is a physical quantity defined as the product of mass multiplied by velocity. Note the definition says velocity, not speed, so momentum is a vector quantity. Its symbol is \mathbf{p} ; here's the equation for momentum:

$$\mathbf{p} = m\mathbf{v}$$

Note that \mathbf{p} is always in the same direction as \mathbf{v} because m is a *scalar value* (that is, a single value, not a value with multiple components like a vector). It turns out that momentum is conserved in collisions, which means that the momentum before a collision is the same as the momentum after a collision. So if you know the original momentum in the collision, you can predict what the situation will be after the collision (and physicists are always delighted by such predictions).

What are the units of momentum? Momentum is mass times velocity, so its unit is kilogram · meters/second (kg·m/s) in the MKS system.



Q. Suppose that you're in an 800.0-kg race car going 200.0 miles an hour due east. If you have a mass of 60.0 kg, what is the total momentum?

EXAMPLE

A. The correct answer is 7.7×10^4 kg·m/s, due east.

1. Use the equation $\mathbf{p} = m\mathbf{v}$.
2. Plug in the numbers, after first converting 200 miles per hour to 89.4 m/s:

$$\mathbf{p} = m\mathbf{v} = (800.0 \text{ kg} + 60.0 \text{ kg})(89.4 \text{ m/s}) = 7.7 \times 10^4 \text{ kg}\cdot\text{m/s, due east.}$$

3 You're running north at 3.0 m/s. If you have a mass of 80.0 kg, what is your momentum?

4 You're falling out of an airplane, and before opening your parachute, you hit a speed of 100.0 m/s. What is your momentum if you have a mass of 80.0 kg?

5 You're pushing a 10.0-kg box of dishes across the kitchen floor at a rate of 4.0 m/s. What is its momentum?

6 You're pushing an 800.0-kg car down the road, and it's going at 6.0 m/s west. How much momentum does it have?

Relating Impulse and Momentum

It turns out that there's a direct connection between impulse and momentum. If you hit a pool ball with a cue, the cue imparts a certain impulse to the ball, causing the ball to end up with a particular momentum.

How can you relate impulse to momentum? Easy. The impulse you impart to an object gives it a change in momentum equal to that impulse, so

$$\text{Impulse} = Ft = \Delta p = m\Delta v$$



Q. If you hit a stationary 160.0-g pool ball with a force of 100.0 N for 0.10 seconds, what is its final speed?

EXAMPLE

A. The correct answer is 63 m/s.

1. Use the equation $Ft = \Delta p$.

2. Find the impulse first:

$$\text{Impulse} = Ft = (100.0 \text{ N})(0.10 \text{ s}) = 10 \text{ N}\cdot\text{s}$$

3. That impulse is the change in the pool ball's momentum:

$$\Delta p = (100.0 \text{ N})(0.10 \text{ s}) = 10 \text{ kg}\cdot\text{m/s}$$

4. Now the pool ball's initial momentum was zero, so the final momentum is:

$$p = 0 + \Delta p = 10 \text{ kg}\cdot\text{m/sec}$$

5. Momentum equals mass times velocity, so to find speed (the magnitude of velocity), you can solve for v , like so:

$$\begin{aligned}v &= \frac{p}{m} \\ &= \frac{10 \text{ kg}\cdot\text{m/s}}{0.16 \text{ kg}} = 63 \text{ m/s}\end{aligned}$$

7 You hit a hockey puck, mass 170 g, with a force of 150 N for 0.10 seconds. If it started at rest, what is its final speed?

8 You're standing on an ice rink when another skater hits you, imparting a force of 200.0 N for 0.20 seconds. If you have a mass of 90.0 kg, what is your final speed?

9 You kick a 450-g soccer ball with a force of 400.0 N for 0.20 seconds. What is its final speed?

10 You hit a 150-g baseball with a force of 400.0 N for 0.10 seconds. The baseball was traveling toward you at 40 m/s. What is its final speed?

Conserving Momentum

The major tool you have in calculating what's going to happen in collisions is the knowledge that momentum is conserved. You know that the total momentum before the collision is the same as the total momentum after the collision as long as there are no significant outside forces.

When you have two objects that collide (one is initially at rest and the other is moving), and you know the final velocity and mass of one object after the collision, you can calculate the final velocity of the other object. You can do this because the total momentum is conserved, so it's the same, before and after the collision, as shown by this equation:

$$P_o = P_f$$



EXAMPLE

Q. A pool ball with a mass of 170 g and a speed of 30 m/s hits another pool ball that's at rest. If the first pool ball ends up going in the same direction with a speed of 10 m/s, what is the new speed of the second pool ball?

A. The correct answer is 20 m/s.

1. Use the equation $P_o = P_f$.
2. All travel is in the same direction in this question, so you can treat it as a scalar equation. Find the original total momentum:

$$p_o = (0.17 \text{ kg})(30 \text{ m/s}) = 5.1 \text{ kg}\cdot\text{m/s}$$

3. The original total momentum equals the total final momentum, which is given by this expression, where p_{2f} is the final momentum of the second ball:

$$p_o = 5.1 \text{ kg}\cdot\text{m/s} = p_f = (0.17 \text{ kg})(10 \text{ m/s}) + p_{2f}$$

4. Solve for p_{2f} :

$$p_{2f} = 5.1 \text{ kg}\cdot\text{m/s} - 1.7 \text{ kg}\cdot\text{m/s} = 3.4 \text{ kg}\cdot\text{m/s}$$

5. Divide p_{2f} by the pool ball's mass to find its speed:

$$\frac{p_{2f}}{m} = \frac{3.4 \text{ kg}\cdot\text{m/s}}{0.17 \text{ kg}} = 20 \text{ m/s}$$

11 A 160-g hockey puck traveling at 60 m/s hits a stationary puck. If the first puck ends up going at 20 m/s in the same direction as the second puck, how fast is the second puck moving?

12 You're driving a bumper car at an amusement park circus at 18 m/s, and you hit another car that's at rest. If you end up going at 6.0 m/s, what is the final speed of the other car, given that both cars have 100.0-kg mass, you have 80.0-kg mass, and the other person has a mass of 70.0 kg?

13 On the athletic field, a golf ball with a mass of 0.20 kg and speed of 100.0 m/s hits a soccer ball at rest that has a mass of 0.45 kg. If the golf ball ends up at rest, what is the soccer ball's final speed, given that it travels in the same direction as the golf ball was originally traveling?

14 You're stopped at a traffic light when a 1,000.0-kg car (including driver) hits you from behind at 10.0 m/s. Ouch! If the other car ends up moving at 6 m/s, and if you and your car have a mass of 940 kg, what is your final speed?

Conserving Kinetic Energy — or Not

In some types of collisions, called *elastic collisions*, kinetic energy and momentum are conserved. What do elastic collisions look like? In general, there's no deformation of any of the objects from an elastic collision. Here are the equations for the conservation of these factors:

$$KE_o = KE_f$$

$$\mathbf{p}_o = \mathbf{p}_f$$

Check out this idea in action: Suppose that you're in a car when you hit the car in front of you (elastically — no deformation of bumpers is involved), which started at rest. You know that momentum is always conserved, and you know that the car in front of you was stopped when you hit it, so if your car is car 1 and the other one is car 2, you get this equation:

$$m_1\mathbf{v}_{f1} + m_2\mathbf{v}_{f2} = m_1\mathbf{v}_{o1}$$

This equation can't tell you what \mathbf{v}_{f1} and \mathbf{v}_{f2} are, because there are two unknowns and only one equation. You can't solve for either \mathbf{v}_{f1} or \mathbf{v}_{f2} exactly in this case, even if you know the masses and \mathbf{v}_{o1} . So to solve for both final speeds, you need another equation to constrain what's going on here. That means using the conservation of kinetic energy. The collision was an elastic one, so kinetic energy was indeed conserved. That means that

$$\frac{1}{2}m_1\mathbf{v}_{f1}^2 + \frac{1}{2}m_2\mathbf{v}_{f2}^2 = \frac{1}{2}m_1\mathbf{v}_{o1}^2$$

With two equations and two unknowns, \mathbf{v}_{f1} and \mathbf{v}_{f2} , you can solve for those unknowns in terms of the masses and \mathbf{v}_{o1} .

You probably won't be asked to solve questions of this kind on physics tests because, in addition to being two simultaneous equations, the second equation has a lot of squared velocities in it. But it's one you may see in homework. When you do the math, you get

$$v_{f1} = \frac{(m_1 - m_2)v_{o1}}{(m_1 + m_2)}$$

and

$$v_{f2} = \frac{2 \cdot m_1 \cdot v_{o1}}{(m_1 + m_2)}$$

This is a more substantial result than you get from problems that just use the conservation of momentum; in such problems, you can solve for only one final speed. Here, using both the conservation of momentum and kinetic energy, you can solve for both objects' final speeds. However, remember that the formulas in this section work only in the special case of an elastic collision. Also, the formulas would need to be modified if the initial velocity of the second object wasn't zero.



EXAMPLE

Q. You're in a car that hits the at-rest car in front of you. If you and your car's mass is 1,000.0 kg, the mass of the car and driver ahead of you is 900.0 kg, and if you started at 44 m/s, what are the final speeds of the two cars? Assume that the collision is elastic and all the action happens in the same line as your original direction of travel.

A. The correct answer is that your car moves 2.3 m/s, and the other car moves 46 m/s.

1. You know that this collision is elastic and the second car starts at rest, so you can use the equations given earlier in this section. Use this equation to find the final speed of your car:

$$v_{f1} = \frac{(m_1 - m_2)v_{o1}}{(m_1 + m_2)}$$

2. Plug in the numbers:

$$v_{f1} = \frac{(m_1 - m_2)v_{o1}}{(m_1 + m_2)} = \frac{(100.0 \text{ kg})(44 \text{ m/s})}{1,900.0 \text{ kg}} = 2.3 \text{ m/s}$$

3. Use this equation to find the final speed of the other car:

$$v_{f2} = \frac{2m_1v_{o1}}{(m_1 + m_2)}$$

4. Plug in the numbers:

$$v_{f2} = \frac{2m_1v_{o1}}{(m_1 + m_2)} = \frac{2(1,000.0 \text{ kg})(44 \text{ m/s})}{(1,900.0 \text{ kg})} = 46 \text{ m/s}$$

15 A 160-g hockey puck traveling at 60.0 m/s hits a stationary puck with the same mass. What are the final speeds of the pucks, given that the collision is elastic and that all motion takes place along the same line?

16 You're driving a bumper car at 23 m/s, and you hit another bumper car that's at rest. If you and your car have a mass of 300 kg, and the mass of the other car and driver is 240 kg, what are the final speeds of the cars?

Collisions in Two Dimensions

Collisions can take place in two dimensions. For example, soccer balls can move any which way on a soccer field, not just along a single line. Soccer balls can end up going north or south, east or west, or a combination of those. So you have to be prepared to handle collisions in two dimensions.



EXAMPLE

Q. In Figure 8-1, there's been an accident at an Italian restaurant, and two meatballs are colliding. Assuming that $v_{o1} = 10.0$ m/s, $v_{o2} = 5.0$ m/s, $v_{f2} = 6.0$ m/s, and the masses of the meatballs are equal, what are θ and v_{f1} ?

A. The correct answer is $\theta = 24^\circ$ and $v_{f1} = 8.2$ m/s.

1. You can't assume that these meatballs conserve kinetic energy when they collide because the meatballs probably deform from the collision. However, momentum is conserved. In fact, momentum is conserved in both the x and y directions, which means

$$p_{fx} = p_{ox}$$

and

$$p_{fy} = p_{oy}$$

2. Here's what the original momentum in the x direction was:

$$p_{fx} = p_{ox} = m_1 v_{o1} \cos 40^\circ + m_2 v_{o2}$$

3. Momentum is conserved in the x direction, so you get

$$p_{fx} = p_{ox} = m_1 v_{o1} \cos 40^\circ + m_2 v_{o2} = m_1 v_{f1x} + m_2 v_{f2} \cos 30^\circ$$

4. Which means that

$$m_1 v_{f1x} = m_1 v_{o1} \cos 40^\circ + m_2 v_{o2} - m_2 v_{f2} \cos 30^\circ$$

5. Divide by m_1 :

$$v_{f1x} = \frac{m_1 v_{o1} \cos 40^\circ + m_2 v_{o2} - m_2 v_{f2} \cos 30^\circ}{m_1}$$

And because $m_1 = m_2$, this becomes

$$v_{f1x} = v_{o1} \cos 40^\circ + v_{o2} - v_{f2} \cos 30^\circ$$

6. Plug in the numbers:

$$\begin{aligned} v_{f1x} &= v_{o1} \cos 40^\circ + v_{o2} - v_{f2} \cos 30^\circ \\ &= (10 \text{ m/s})(0.766) + 5.0 \text{ m/s} - (6.0 \text{ m/s})(0.866) \\ &= 7.5 \text{ m/s} \end{aligned}$$

7. Now for the y direction. Here's what the original momentum in the y direction looks like (in the downward direction):

$$p_{fy} = p_{oy} = m_1 v_{o1} \sin 40^\circ$$

8. Set that equal to the final momentum in the y direction:

$$p_{fy} = p_{oy} = m_1 v_{o1} \sin 40^\circ = m_1 v_{f1y} + m_2 v_{f2} \sin 30^\circ$$

That equation turns into:

$$m_1 v_{f1y} = m_1 v_{o1} \sin 40^\circ - m_2 v_{f2} \sin 30^\circ$$

9. Solve for the final velocity component of meatball 1's y velocity:

$$v_{f1y} = \frac{m_1 v_{o1} \sin 40^\circ - m_2 v_{f2} \sin 30^\circ}{m_1}$$

10. Because the two masses are equal, this becomes

$$v_{f1y} = v_{o1} \sin 40^\circ - v_{f2} \sin 30^\circ$$

11. Plug in the numbers:

$$\begin{aligned} v_{f1y} &= v_{o1} \sin 40^\circ - v_{f2} \sin 30^\circ \\ &= (10 \text{ m/s})(0.642) - (6.0 \text{ m/s})(0.5) \\ &= 3.4 \text{ m/s} \end{aligned}$$

12. So:

$$\begin{aligned} v_{f1x} &= 7.5 \text{ m/s (to the right)} \\ v_{f1y} &= 3.4 \text{ m/s (downward)} \end{aligned}$$

That means that the angle θ is

$$\begin{aligned} \theta &= \tan^{-1} \left(\frac{3.4 \text{ m/s}}{7.5 \text{ m/s}} \right) \\ &= \tan^{-1}(0.45) \\ &= 24^\circ \end{aligned}$$

And the magnitude of v_{f1} is

$$\begin{aligned} v_{f1} &= \sqrt{v_{f1x}^2 + v_{f1y}^2} \\ &= \sqrt{(7.5 \text{ m/s})^2 + (3.4 \text{ m/s})^2} \\ &= 8.2 \text{ m/s} \end{aligned}$$

- 17 Assume that the two objects in Figure 8-1 are hockey pucks of equal mass. Assuming that $v_{o1} = 15 \text{ m/s}$, $v_{o2} = 7.0 \text{ m/s}$, and $v_{f2} = 7.0 \text{ m/s}$, what are θ and v_{f1} , assuming that momentum is conserved but kinetic energy is not?

- 18 Assume that the two objects in Figure 8-2 are tennis balls of equal mass. Assuming that $v_{o1} = 12 \text{ m/s}$, $v_{o2} = 8.0 \text{ m/s}$, and $v_{f2} = 6.0 \text{ m/s}$, what are θ and v_{f1} , assuming that momentum is conserved but kinetic energy is not?

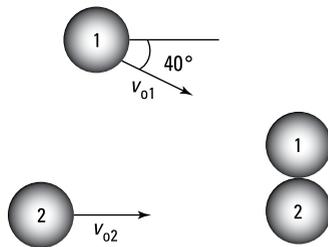
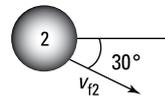
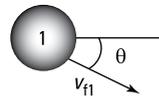


FIGURE 8-1:
Colliding
objects.



© John Wiley & Sons, Inc.

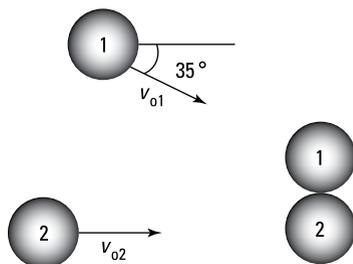
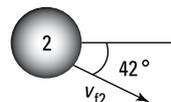
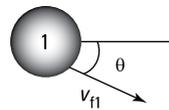


FIGURE 8-2:
A collision
between tennis
balls.



© John Wiley & Sons, Inc.

Answers to Problems about Momentum and Kinetic Energy

The following are the answers to the practice questions presented in this chapter. You see how to work out each answer, step by step.

1 **10 N-s**

1. Use the equation **Impulse = Ft**.
2. Plug in the numbers:

$$\text{Impulse} = Ft = (100.0 \text{ N})(0.1 \text{ s}) = 10 \text{ N-s}$$

2 **30 N-s**

1. Use the equation **Impulse = Ft**.
2. Plug in the numbers:

$$\text{Impulse} = Ft = (300.0 \text{ N})(0.10 \text{ s}) = 30 \text{ N-s}$$

3 **240 kg-m/s north**

1. Use the equation **p = mv**.
2. Plug in the numbers:

$$\mathbf{p} = mv = (80.0 \text{ kg})(3.0 \text{ m/s}) = 240 \text{ kg-m/s north}$$

4 **8,000 kg-m/s downward**

1. Use the equation **p = mv**.
2. Plug in the numbers:

$$\mathbf{p} = mv = (80.0 \text{ kg})(100.0 \text{ m/s}) = 8,000 \text{ kg-m/s downward}$$

5 **40 kg-m/s**

1. Use the equation **p = mv**.
2. Plug in the numbers:

$$\mathbf{p} = mv = (10.0 \text{ kg})(40.0 \text{ m/s}) = 40 \text{ kg-m/s}$$

6 **4,800 kg-m/s west**

1. Use the equation **p = mv**.
2. Plug in the numbers:

$$\mathbf{p} = mv = (800.0 \text{ kg})(6.0 \text{ m/s}) = 4,800 \text{ kg-m/s west}$$

7 **88 m/s**

1. Use the equation **Ft = Δp**.
2. Find the impulse:

$$\text{Impulse} = Ft = (150 \text{ N})(0.10 \text{ s}) = 15 \text{ N-s}$$

3. Because the initial momentum of the puck was zero, that impulse becomes the puck's new momentum:

$$p = (150 \text{ N})(0.10 \text{ s}) = 15 \text{ kg}\cdot\text{m/s}$$

4. Momentum equals mass times velocity, so to find speed (the magnitude of velocity), you solve for v :

$$\begin{aligned}v &= \frac{p}{m} \\&= \frac{15 \text{ kg}\cdot\text{m/s}}{0.17 \text{ kg}} \\&= 88 \text{ m/s}\end{aligned}$$

8 **0.44 m/s**

1. Use the equation $Ft = \Delta p$.
2. Find the impulse:

$$\text{Impulse} = Ft = (200.0 \text{ N})(0.20 \text{ s}) = 40 \text{ N}\cdot\text{s}$$

3. Because your initial momentum was zero, that impulse becomes your new momentum:

$$p = (200.0 \text{ N})(0.20 \text{ s}) = 40 \text{ kg}\cdot\text{m/s}$$

4. Momentum equals mass times velocity, so to find speed (the magnitude of velocity), you solve for v :

$$\begin{aligned}v &= \frac{p}{m} \\&= \frac{40 \text{ kg}\cdot\text{m/s}}{90.0 \text{ kg}} \\&= 0.44 \text{ m/s}\end{aligned}$$

9 **178 m/s**

1. Use the equation $Ft = \Delta p$.
2. Find the impulse:

$$\text{Impulse} = Ft = (400.0 \text{ N})(0.20 \text{ s}) = 80 \text{ N}\cdot\text{s}$$

3. That impulse becomes the soccer ball's new momentum:

$$p = (400.0 \text{ N})(0.20 \text{ s}) = 80 \text{ kg}\cdot\text{m/s}$$

4. Momentum equals mass times velocity, so to find speed (the magnitude of velocity), you solve for v :

$$\begin{aligned}v &= \frac{p}{m} \\&= \frac{80 \text{ kg}\cdot\text{m/s}}{0.45 \text{ kg}} \\&= 178 \text{ m/s}\end{aligned}$$

10 227 m/s

1. Use the equation $Ft = \Delta p$.
2. Find the impulse first:

$$\text{Impulse} = Ft = (400.0 \text{ N})(0.10 \text{ s}) = 40 \text{ N}\cdot\text{s}$$

3. That impulse becomes the ball's change in momentum:

$$\Delta p = (400.0 \text{ N})(0.10 \text{ s}) = 40.0 \text{ kg}\cdot\text{m/s}$$

4. The baseball started with a momentum of $(0.150 \text{ kg})(40 \text{ m/s}) = 6 \text{ kg}\cdot\text{m/s}$ (toward you), so its final momentum will be $40 \text{ kg}\cdot\text{m/s} - 6 \text{ kg}\cdot\text{m/s} = 34 \text{ kg}\cdot\text{m/s}$ (away from you).
5. Momentum equals mass times velocity, so to find speed (the magnitude of velocity), you solve for v :

$$\begin{aligned} v &= \frac{p}{m} \\ &= \frac{34 \text{ kg}\cdot\text{m/s}}{0.150 \text{ kg}} \\ &= 227 \text{ m/s} \end{aligned}$$

11 58 m/s

1. Use the equation $\mathbf{p}_o = \mathbf{p}_f$.
2. The pucks travel in the same direction here, so you can treat this as a scalar equation. Find the original total momentum:

$$p_o = (0.16 \text{ kg})(60 \text{ m/s}) = 9.6 \text{ kg}\cdot\text{m/s}$$

3. This equals the total final momentum, which is given by this expression, where p_{2f} is the final momentum of the second puck:

$$p_o = 9.6 \text{ kg}\cdot\text{m/s} = p_f = (0.16 \text{ kg})(20 \text{ m/s}) + p_{2f}$$

4. Solve for p_{2f} :

$$p_{2f} = 9.6 \text{ kg}\cdot\text{m/s} - 0.32 \text{ kg}\cdot\text{m/s} = 9.28 \text{ kg}\cdot\text{m/s}$$

5. Divide p_{2f} by the puck's mass to find its speed:

$$\frac{p_{2f}}{m} = \frac{9.28 \text{ kg}\cdot\text{m/s}}{0.16 \text{ kg}} = 58 \text{ m/s}$$

12 13 m/s

1. Use the equation $\mathbf{p}_o = \mathbf{p}_f$.
2. The bumper cars travel in the same direction here, so you can treat this as a scalar equation. Find the original total momentum:

$$p_o = (18 \text{ m/s})(100.0 \text{ kg} + 80.0 \text{ kg}) = 3,240 \text{ kg}\cdot\text{m/s}$$

3. This equals the total final momentum, which is given by this expression, where p_{2f} is the final momentum of the second bumper car:

$$p_o = 3,240 \text{ kg}\cdot\text{m/s} = p_f = (100.0 \text{ kg} + 80.0 \text{ kg})(6.0 \text{ m/s}) + p_{2f}$$

4. Solve for p_{2f} :

$$p_{2f} = 3,240 \text{ kg}\cdot\text{m/s} - 1,080 \text{ kg}\cdot\text{m/s} = 2,160 \text{ kg}\cdot\text{m/s}$$

5. Divide p_{2f} by the bumper car and person's mass to find its speed:

$$\begin{aligned}\frac{p_{2f}}{m} &= \frac{2,160 \text{ kg}\cdot\text{m/s}}{100.0 \text{ kg} + 70.0 \text{ kg}} \\ &= 12.7 \text{ m/s}\end{aligned}$$

which rounds to 13 with significant figures

13 **44 m/s**

1. Use the equation $p_o = p_f$.

2. Find the original total momentum:

$$p_o = (0.20 \text{ kg})(100.0 \text{ m/s}) = 20 \text{ kg}\cdot\text{m/s}$$

3. This equals the total final momentum, which is given by this expression, where p_{2f} is the final momentum of the soccer ball:

$$p_o = 20 \text{ kg}\cdot\text{m/s} = p_f = p_{2f}$$

4. Solve for p_{2f} :

$$p_{2f} = 20 \text{ kg}\cdot\text{m/s}$$

5. Divide p_{2f} by the soccer ball's mass to find its speed:

$$\frac{p_{2f}}{m} = \frac{20 \text{ kg}\cdot\text{m/s}}{0.45 \text{ kg}} = 44 \text{ m/s}$$

14 **4.2 m/s**

1. Use the equation $p_o = p_f$.

2. Find the original total momentum:

$$p_o = (1,000.0 \text{ kg})(10.0 \text{ m/s}) = 10,000 \text{ kg}\cdot\text{m/s}$$

3. This equals the total final momentum, which is given by the following expression, where p_{2f} is the final momentum of the car:

$$p_o = 10,000 \text{ kg}\cdot\text{m/s} = p_f = (1,000.0 \text{ kg})(6 \text{ m/s}) + p_{2f}$$

4. Solve for p_{2f} :

$$p_{2f} = 4,000 \text{ kg}\cdot\text{m/s}$$

5. Divide p_{2f} by you and your car's mass to find its speed:

$$\begin{aligned}\frac{p_{2f}}{m} &= \frac{4,000 \text{ kg}\cdot\text{m/s}}{940 \text{ kg}} \\ &= 4.2 \text{ m/s, about 9 mph}\end{aligned}$$

15 **0, 60 m/s**

1. Use this equation to find the final speed of the first puck:

$$v_{f1} = \frac{(m_1 - m_2)v_{o1}}{(m_1 + m_2)}$$

2. Substituting the numbers gives you:

$$v_{f1} = \frac{(m_1 - m_2)v_{o1}}{(m_1 + m_2)} = \frac{(160 \text{ g} - 160 \text{ g})(60 \text{ m/s})}{(160 \text{ g} + 160 \text{ g})} = 0 \text{ m/s}$$

3. Use this equation to find the final speed of the second puck:

$$v_{f2} = \frac{2m_1v_{o1}}{(m_1 + m_2)}$$

4. Putting in the numbers gives you:

$$v_{f2} = \frac{2m_1v_{o1}}{(m_1 + m_2)} = \frac{2(160 \text{ g})(60 \text{ m/s})}{(160 \text{ g} + 160 \text{ g})} = 60 \text{ m/s}$$

Note that when the masses are the same, the first puck stops, and the second puck takes off with the same speed as the first puck had.

16 **You: 2.6 m/s; the other car: 26 m/s**

1. Use this equation to find the final speed of your car:

$$v_{f1} = \frac{(m_1 - m_2)v_{o1}}{(m_1 + m_2)}$$

2. Plug in the numbers:

$$v_{f1} = \frac{(m_1 - m_2)v_{o1}}{(m_1 + m_2)} = \frac{(300 \text{ kg} - 240 \text{ kg})(23 \text{ m/s})}{(300 \text{ kg} + 240 \text{ kg})} = 2.6 \text{ m/s}$$

3. Use this equation to find the final speed of the second car:

$$v_{f2} = \frac{2m_1v_{o1}}{(m_1 + m_2)}$$

4. Plug in the numbers:

$$v_{f2} = \frac{2m_1v_{o1}}{(m_1 + m_2)} = \frac{2(300 \text{ kg})(23 \text{ m/s})}{(300 \text{ kg} + 240 \text{ kg})} = 26 \text{ m/s}$$

17 **14 m/s, 26°**

1. Momentum is conserved in this collision. In fact, momentum is conserved in both the x and y directions, which means the following are true:

$$p_{fx} = p_{ox}$$

$$p_{fy} = p_{oy}$$

2. The original momentum in the x direction was

$$p_{fx} = p_{ox} = m_1v_{o1} \cos 40^\circ + m_2v_{o2}$$

3. Momentum is conserved in the x direction, so

$$p_{fx} = p_{ox} = m_1 v_{o1} \cos 40^\circ + m_2 v_{o2} = m_1 v_{f1x} + m_2 v_{f2} \cos 30^\circ$$

4. Solving for $m_1 v_{f1x}$ gives you:

$$m_1 v_{f1x} = m_1 v_{o1} \cos 40^\circ + m_2 v_{o2} - m_2 v_{f2} \cos 30^\circ$$

5. Divide by m_1 :

$$v_{f1x} = \frac{m_1 v_{o1} \cos 40^\circ + m_2 v_{o2} - m_2 v_{f2} \cos 30^\circ}{m_1}$$

Because $m_1 = m_2$, that equation becomes

$$v_{f1x} = v_{o1} \cos 40^\circ + v_{o2} - v_{f2} \cos 30^\circ$$

6. Plug in the numbers:

$$\begin{aligned} v_{f1x} &= v_{o1} \cos 40^\circ + v_{o2} - v_{f2} \cos 30^\circ \\ &= (15 \text{ m/s})(0.766) + 7.0 \text{ m/s} - (7.0 \text{ m/s})(0.866) \\ &= 12.4 \text{ m/s} \end{aligned}$$

7. Now for the y direction. The original momentum in the y direction was

$$p_{fy} = p_{oy} = m_1 v_{o1} \sin 40^\circ$$

8. Set that equal to the final momentum in the y direction:

$$p_{fy} = p_{oy} = m_1 v_{o1} \sin 40^\circ = m_1 v_{f1y} + m_2 v_{f2} \sin 30^\circ$$

Which turns into

$$m_1 v_{f1y} = m_1 v_{o1} \sin 40^\circ - m_2 v_{f2} \sin 30^\circ$$

9. Solve for the final velocity component of puck 1's y velocity:

$$v_{f1y} = \frac{m_1 v_{o1} \sin 40^\circ - m_2 v_{f2} \sin 30^\circ}{m_1}$$

10. Because the two masses are equal, the equation becomes

$$v_{f1y} = v_{o1} \sin 40^\circ - v_{f2} \sin 30^\circ$$

11. Plug in the numbers:

$$\begin{aligned} v_{f1y} &= v_{o1} \sin 40^\circ - v_{f2} \sin 30^\circ \\ &= (15 \text{ m/s})(0.642) - (7.0 \text{ m/s})(0.5) \\ &= 6.1 \text{ m/s} \end{aligned}$$

12. So

$$\begin{aligned} v_{f1x} &= 12.4 \text{ m/s} \\ v_{f1y} &= 6.1 \text{ m/s} \end{aligned}$$

That means the angle θ is

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{6.1 \text{ m/s}}{12.4 \text{ m/s}}\right) \\ &= \tan^{-1}(0.49) = 26^\circ\end{aligned}$$

And the magnitude of v_{f1} is

$$\begin{aligned}v_{f1} &= \sqrt{v_{f1x}^2 + v_{f1y}^2} \\ &= \sqrt{(12.4 \text{ m/s})^2 + (6.1 \text{ m/s})^2} \\ &= 14 \text{ m/s}\end{aligned}$$

18 **14 m/s, 12°**

1. In this situation, momentum is conserved in both the x and y directions, so the following are true:

$$p_{fx} = p_{ox}$$

$$p_{fy} = p_{oy}$$

2. The original momentum in the x direction was

$$p_{fx} = p_{ox} = m_1 v_{o1} \cos 35^\circ + m_2 v_{o2}$$

3. Momentum is conserved in the x direction, so:

$$p_{fx} = p_{ox} = m_1 v_{o1} \cos 35^\circ + m_2 v_{o2} = m_1 v_{f1x} + m_2 v_{f2} \cos 42^\circ$$

4. Which means:

$$m_1 v_{f1x} = m_1 v_{o1} \cos 35^\circ + m_2 v_{o2} - m_2 v_{f2} \cos 42^\circ$$

5. Divide by m_1 :

$$v_{f1x} = \frac{m_1 v_{o1} \cos 35^\circ + m_2 v_{o2} - m_2 v_{f2} \cos 42^\circ}{m_1}$$

Because $m_1 = m_2$, this becomes

$$v_{f1x} = v_{o1} \cos 35^\circ + v_{o2} - v_{f2} \cos 42^\circ$$

6. Plug in the numbers:

$$\begin{aligned}v_{f1x} &= v_{o1} \cos 35^\circ + v_{o2} - v_{f2} \cos 42^\circ \\ &= (12 \text{ m/s})(0.82) + 8.0 \text{ m/s} - (6.0 \text{ m/s})(0.73) \\ &= 13.4 \text{ m/s}\end{aligned}$$

7. Now for the y direction. The original momentum in the y direction was

$$p_{fy} = p_{oy} = m_1 v_{o1} \sin 35^\circ$$

8. Set that equal to the final momentum in the y direction:

$$p_{fy} = p_{oy} = m_1 v_{o1} \sin 35^\circ = m_1 v_{f1y} + m_2 v_{f2} \sin 42^\circ$$

Solving for $m_1 v_{f1y}$ gives you:

$$m_1 v_{f1y} = m_1 v_{o1} \sin 35^\circ - m_2 v_{f2} \sin 42^\circ$$

9. Solve for the final velocity component of puck 1's y velocity:

$$v_{f1y} = \frac{m_1 v_{o1} \sin 35^\circ - m_2 v_{f2} \sin 42^\circ}{m_1}$$

10. Because the two masses are equal, the equation becomes

$$v_{f1y} = v_{o1} \sin 35^\circ - v_{f2} \sin 42^\circ$$

11. Plug in the numbers:

$$\begin{aligned} v_{f1y} &= v_{o1} \sin 35^\circ - v_{f2} \sin 42^\circ \\ &= (12 \text{ m/s})(0.57) - (6.0 \text{ m/s})(0.67) \\ &= 2.9 \text{ m/s} \end{aligned}$$

12. So:

$$\begin{aligned} v_{f1x} &= 13.4 \text{ m/s} \\ v_{f1y} &= 2.9 \text{ m/s} \end{aligned}$$

Which means the angle θ is

$$\begin{aligned} \theta &= \tan^{-1} \left(\frac{2.9 \text{ m/s}}{13.4 \text{ m/s}} \right) \\ &= \tan^{-1}(0.22) = 12^\circ \end{aligned}$$

And the magnitude of v_{f1} is

$$\begin{aligned} v_{f1} &= \sqrt{v_{f1x}^2 + v_{f1y}^2} \\ &= \sqrt{(13.4 \text{ m/s})^2 + (2.9 \text{ m/s})^2} \\ &= 14 \text{ m/s} \end{aligned}$$

IN THIS CHAPTER

- » Finding tangential speed and acceleration
- » Getting angular velocity
- » Calculating torque
- » Working with rotational kinematics
- » Handling rotational equilibrium

Chapter 9

Winding It Up: Rotational Motion and Torque

When things rotate, lots of physics happens. If you have a merry-go-round, for example, and apply a force to its edge, what happens next is explained by physics.

This chapter covers tangential speed, tangential acceleration, angular velocity and angular acceleration treated as a vector, and torque. The practice problems here will help you become a master at handling these types of physics problems.

Finding Tangential Speed

Tangential speed is the magnitude of tangential velocity. Take a look at Figure 9-1, where you see a ball revolving around the origin. As it sweeps around the origin with linear speed v (which keeps changing direction as the ball moves in a circle), the angle θ increases in time.

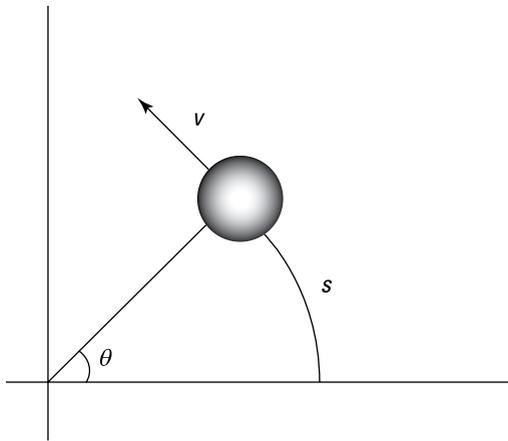


FIGURE 9-1:
A rotating ball.

© John Wiley & Sons, Inc.

How do you relate the distance the ball has traveled, s , to the angle θ ? If you measure θ in radians, that relationship is the following, where r is the radius of the circle:

$$s = r\theta$$

Also, you know that:

$$v = \frac{s}{t}$$

That means that you can substitute for s to get:

$$v = \frac{s}{t} = \frac{r\theta}{t}$$

And $\omega = \theta/t$, which means:

$$v = \frac{s}{t} = \frac{r\theta}{t} = r\omega$$

So

$$v = r\omega$$



REMEMBER

In this case, v is called tangential velocity — that is, it's the instantaneous linear velocity of the ball as it goes around in the circle. Tangential velocity is always perpendicular to the radius of the circle and is in the direction of travel of the object going around the circle.



Q. A ball on a string is going around in a circle at 6.0 radians/s. What is its tangential velocity if the radius of the circle is 2.0 m?

EXAMPLE

A. The correct answer is 12 m/s.

1. Use the equation $v = r\omega$.
2. Plug in the numbers:

$$v = r\omega = (2.0 \text{ m})(6.0 \text{ radians/s}) = 12 \text{ m/s}$$

1 If a satellite is orbiting Earth, which has an average radius of 3,960 miles, at an altitude of 150 miles and an angular speed of 1.17×10^{-3} radians/s, what is the satellite's tangential speed in mph?

2 You're flying a toy plane on a string, and it's going around at 20.0 mph, 100.0 feet from you. What is its angular speed in radians/s?

3 A racing car is going around a circular track with a 400.0-ft radius at 50.0 mph. What is its angular speed in radians/s?

4 The tip of an airplane propeller is going at 500.0 mph. If the propeller has a radius of 3.0 ft, what is its angular speed?

Targeting Tangential Acceleration

Besides tangential velocity, you can have tangential acceleration. For instance, if you start a helicopter's rotors, the tip of any rotor starts with a tangential velocity of zero and increases with time. Because the velocity vector's magnitude increases and its direction changes, there's acceleration, which is expressed like so:

$$a = \frac{\Delta v}{\Delta t}$$

How can you relate this to angular quantities? Because tangential speed is $v = r\omega$, you can plug that into the acceleration equation:

$$a = \frac{\Delta v}{\Delta t} = \frac{\Delta(r\omega)}{\Delta t} = \frac{r\Delta\omega}{\Delta t}$$

where you can write $\Delta(r\omega) = r\Delta\omega$ because r is constant.

And because $\Delta\omega/\Delta t = \alpha$, which is the angular acceleration, this equation becomes

$$a = \frac{\Delta v}{\Delta t} = \frac{\Delta(r\omega)}{\Delta t} = r\alpha$$

Which breaks down to

$$a = r\alpha$$

What this all means is that the tangential acceleration at radius r is $a = r\alpha$.



EXAMPLE

Q. A set of helicopter blades has a radius of 4.3 m. If a point on the tip of one blade starts at 0 m/s and ends up 60 seconds later with a speed of 400 m/s, what was the angular acceleration?

A. The correct answer is 1.6 radians/s².

1. Use this equation:

$$a = \frac{\Delta v}{\Delta t} = r\alpha$$

2. Plug in the numbers:

$$a = \frac{\Delta v}{\Delta t} = \frac{(400 \text{ m/s} - 0 \text{ m/s})}{(60 \text{ s})} = r\alpha$$

3. Divide both sides by r :

$$\frac{(400 \text{ m/s} - 0 \text{ m/s})}{(4.3 \text{ m})(60 \text{ s})} = \alpha$$

4. Do the math:

$$\frac{(400 \text{ m/s} - 0 \text{ m/s})}{(4.3 \text{ m})(60 \text{ s})} = \alpha = 1.6 \text{ radians/s}^2$$

5 If a point on the edge of a tire with a radius of 0.50 m starts at rest and ends up 3.5 minutes later at 88 m/s (about 197 mph), what was the magnitude of its average angular acceleration?

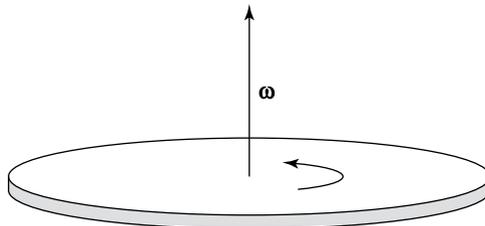
6 You're flying a toy plane on a string, and it's going around at 20.0 m/s, 10.0 m from you. If it accelerates to a final velocity of 30.0 m/s in 80.0 seconds, what is its angular acceleration?

Angular Velocity as a Vector

Angular velocity is really a vector, ω . The question is, which way does it point? Think of it this way: If you have a flying disk being tossed back and forth between two players, it's spinning. So which way can ω point so it stays constant in magnitude and direction?

Take a look at Figure 9-2 for the answer. The ω vector points out of the plane of rotation.

FIGURE 9-2:
Angular
velocity as a
vector.



© John Wiley & Sons, Inc.



You find the direction of the ω vector by wrapping your right hand around in the direction of rotation. Your right thumb will point in the direction of the ω vector.

TIP



Q. A helicopter's blades are rotating in a horizontal plane, and they're going counterclockwise when viewed from above. Which way does ω point?

EXAMPLE

A. The correct answer is upward.

1. Curl your right hand in the direction of rotational motion — counterclockwise.
2. Your right thumb points upward, indicating the direction of the ω vector.

7

Suppose that you're flying a toy plane on a string, and it's going around clockwise as viewed from above. Which way does ω point?

8

Suppose that you're driving forward. Which way does ω point for the left front tire?

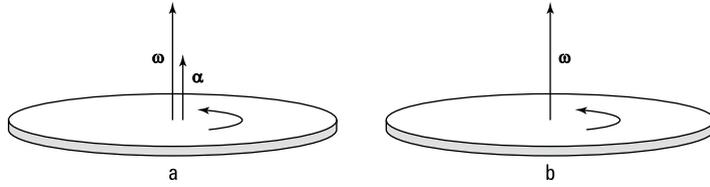
Angular Acceleration as a Vector

Like angular velocity, angular acceleration is a vector; it's represented by the symbol α . But unlike angular velocity, angular acceleration need not be perpendicular to the plane of rotation. The angular acceleration vector just points in the direction of change of the angular velocity vector.

Figure 9-3 shows angular acceleration in the same direction as the angular velocity vector. That means the angular velocity vector will grow in time.

Bear in mind that α need not be perpendicular to the rotation — it just points in the direction in which ω is changing. For example, if you're turning the wheels of a car, the vector αt points in a direction so when it's added to the original angular velocity, ω_0 , you get the new angular velocity, ω_t .

FIGURE 9-3:
Angular
acceleration as
a vector.



© John Wiley & Sons, Inc.



Q. A helicopter's blades are rotating in a horizontal plane, and they're going counterclockwise when viewed from above. As they slow down, which way does α point?

EXAMPLE

A. The correct answer is downward.

1. Curl your right hand in the direction of rotational motion — counterclockwise.
2. Your right thumb points upward, indicating the direction of the ω vector.
3. The ω vector is decreasing in magnitude with time while staying in the same direction, which means α points in the opposite direction — downward.

9 Suppose that you're flying a toy plane on a string, and it's going around clockwise as viewed from above. In time, the plane is slowing down. Which way does α point?

10 A flying disc tossed from one player to another spins as it flies and slows down. If it's spinning counterclockwise when viewed from above, which way does α point?

Doing the Twist: Torque

Torque, represented by the symbol τ , is the rotational analog of force in physics. Torque is much like force but differs in that it's the amount of twist, not push, that occurs. For example, take a look at the door, viewed from above, in Figure 9-4, where the black dot represents the location of the hinge.

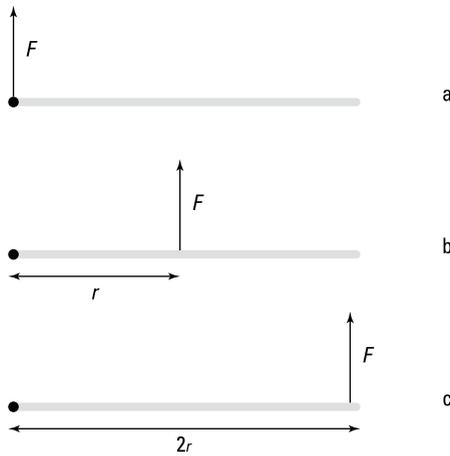


FIGURE 9-4:
Torque at work.

© John Wiley & Sons, Inc.

If you push on the hinge, as shown in Figure 9-4a, the door doesn't open. That's because there's no torque on the door to turn it. On the other hand, if you push on the midpoint of the door, as shown in Figure 9-4b, the door opens. And if you push on the outer edge of the door, it opens more easily (that's Figure 9-4c).

The more force you apply, the more torque there is; the farther out from the turning point (the hinge) you push, the more torque there is. Following is the equation for torque; F is the force you're applying, and l is the lever arm — the perpendicular distance from the axis of rotation to the point where you apply the external force:

$$\tau = lF$$

The units of torque are force multiplied by distance, so that's N·m in the MKS system, dyne·cm in the CGS system, and foot-pounds in the English system.

It's important to realize that the lever arm is the effective distance at which the force acts in a perpendicular direction. What does that mean? In Figure 9-4a, the torque is zero because the lever arm is zero. In Figure 9-4b, the torque is rF because the lever arm is r . In Figure 9-4c, the torque is $(2r)F$ because the lever arm is $2r$.

The lever arm is always perpendicular to the force applied, so how do you find the torque in a situation like the one shown in Figure 9-5, where the force isn't perpendicular to the door?

Here, you have to find the effective perpendicular distance at which the force acts, as shown in Figure 9-5. (Alternatively, you can find the component of the force perpendicular to the door.) That means the torque is the following, where θ is the angle between the force and the door:

$$\tau = rF \sin \theta$$



REMEMBER

Even if you didn't follow that business about lever arms and effective perpendicular distances, remember this equation, because it tells you what the torque is in general. If you apply a force F at a displacement r from a pivot point where the angle between that displacement and F is θ , the torque you produce is $\tau = rF \sin \theta$.

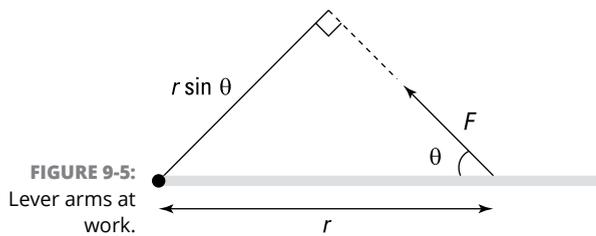


FIGURE 9-5:
Lever arms at
work.

© John Wiley & Sons, Inc.

Torque is a vector, too. Like ω , τ points perpendicular to the plane of motion. If the torque tends to increase the angle θ (which is measured counterclockwise), then the torque points out of the plane of motion and you say that the torque is positive. On the other hand, if the torque tends to decrease the angle θ (that is, if the torque tends to cause clockwise motion), then the torque points into the plane of motion and you say that the torque is negative. In general, the direction of the torque follows the right-hand rule discussed earlier in this section: If you wrap your fingers in the direction that the twist is applied, your thumb points in the direction of the torque.



EXAMPLE

Q. You push a merry-go-round at its edge, perpendicular to the radius. If the merry-go-round has a diameter of 3.0 m, and you push with a force of 200 N, what torque are you applying?

A. The correct answer is 300 N·m.

1. Use the equation $\tau = rF \sin \theta$.

2. Plug in the numbers:

$$\tau = rF \sin \theta = (1.5 \text{ m}) (200 \text{ N}) \sin 90^\circ = 300 \text{ N}\cdot\text{m}$$



REMEMBER

Some problems will give you the radius, and others will give you the diameter, like the example question here. When you're given the diameter, remember to divide the number in half to get the radius. That's where the 1.5 m came from in the preceding equation.

11 You're opening a door by pushing on its outer edge with a force of 100.0 N. If the door is 1.3 m wide, what torque are you applying if you push perpendicular to the door?

12 The hot water tap in your shower is stuck. In anger, you apply 200.0 N of force to the outer edge of the handle, which has a turning radius of 10.0 cm. What torque are you applying?

- 13 You're shoveling snow, holding the shovel handle in your right hand. Your left hand is placed halfway down the shovel's shaft and is providing the lifting motion. If you apply 200.0 N of force at an angle of 78° to the shovel and the shovel is 1.5 m long, how much torque are you applying?

- 14 You apply force on a wrench to loosen a pipe. If the wrench is 25 cm long and you apply 150 N at an angle of 67° with respect to the wrench, what torque are you applying?

The Balancing Act: Rotational Equilibrium

Sometimes you encounter rotational equilibrium problems in physics. In such problems, the torques all balance out, and nothing rotates. For example, take a look at the situation in Figure 9-6, where a ladder with a person on it is balanced against a wall. Will the force of friction keep the ladder from moving, if θ is 45° and the static coefficient of friction with the floor is 0.70?

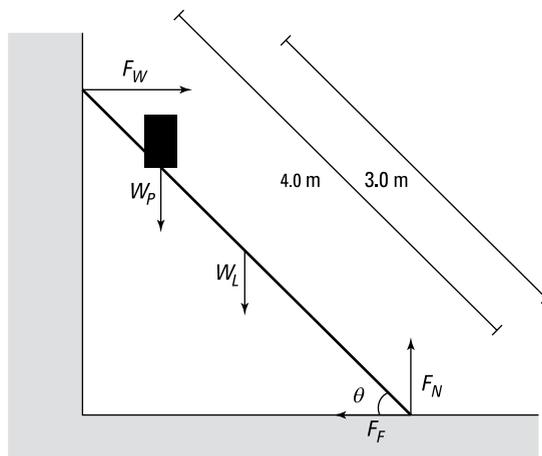


FIGURE 9-6: Checking a ladder.

© John Wiley & Sons, Inc.

This is a rotational equilibrium question: If the sum of the torques is zero ($\Sigma\tau = 0$), the ladder won't fall. But if the force of friction can't supply enough torque, the ladder will indeed rotate and therefore fall.



EXAMPLE

Q. Will the ladder in Figure 9-6 fall? (Assume that the frictional force between the wall and the ladder is insignificant.) Here are the forces involved:

- » F_W = Force exerted by the wall on the ladder
- » W_p = Weight of the person = 450 N
- » W_L = Weight of the ladder = 200 N (you can assume it's concentrated at the ladder's center)
- » F_F = Force of friction holding the ladder in place
- » F_N = Normal force

A. The correct answer is that the ladder will not slip.

1. You want the ladder to be in both linear ($\Sigma F = 0$) and rotational ($\Sigma\tau = 0$) equilibrium. To be in linear equilibrium, the force exerted by the wall on the ladder, F_W , must be the same as the force of friction in magnitude but opposite in direction because those are the only two horizontal forces. So if you can find F_W , you know what the force of friction, F_F , needs to be.
2. To find F_W , take a look at the torques around the bottom of the ladder, using that point as the pivot point. All the torques around that point have to add up to zero. The torque due to the force from the wall against the ladder is

$$-F_W(4.0 \text{ m}) \sin 45^\circ = -2.83 F_W$$

This torque is negative because it tends to produce a clockwise motion toward smaller angles.

3. The torque due to the person's weight is

$$W_p(3.0 \text{ m}) \cos 45^\circ = (450 \text{ N})(3.0 \text{ m})(0.707) = 954 \text{ N}\cdot\text{m}$$

4. The torque due to the ladder's weight is

$$W_L(2.0 \text{ m}) \cos 45^\circ = (200 \text{ N})(2.0 \text{ m})(0.707) = 283 \text{ N}\cdot\text{m}$$

5. Both torques are positive, so because $\Sigma\tau = 0$, you get this result when you add all the torques together:

$$954 \text{ N}\cdot\text{m} + 283 \text{ N}\cdot\text{m} - 2.83 F_W = 0$$

6. Solve for F_W :

$$F_W = \frac{954 \text{ N}\cdot\text{m} + 283 \text{ N}\cdot\text{m}}{2.83 \text{ m}} = 437 \text{ N}$$

That means the force the wall exerts on the ladder is 437 N. Note that is also equal to the frictional force of the bottom of the ladder on the floor because F_w and the frictional force are the only two horizontal forces in the whole system. So the force of friction needed is

$$F_F = 437 \text{ N}$$

7. What is the maximal force of friction possible given the static coefficient of friction between the ladder and the floor? Use this equation:

$$F_{F \text{ maximal}} = \mu_s F_N$$

8. F_N is the normal force of the floor pushing up on the ladder, and it must balance all the downward pointing forces in this problem because of linear equilibrium. That means you have the following:

$$F_N = W_p + W_L = 650 \text{ N}$$

9. Plug in the numbers, using the value of μ_s , 0.70:

$$F_{F \text{ maximal}} = \mu_s F_N = (0.70)(650 \text{ N}) = 455 \text{ N}$$

That's your answer — you need 437 N, and you actually have up to 455 N available, so the ladder isn't going to slip.

15 You're opening a door by pushing on its outer edge with a force of 100 N at 90° , and someone is trying to keep the door shut by pulling on it one-third of the door's width from the hinge. What force does the other person need to supply perpendicular to the door to keep the door in rotational equilibrium?

16 You're on a teeter-totter with a total length of $2L$ with a person twice your weight. The other person's sitting only one-third of the distance from the pivot point, however. Where must you sit to balance the person out?

17 A motor can provide 5,000 N·m of torque. If you can provide 33 N of force, how far away from the motor's axle must you be to produce rotational equilibrium?

18 The cap on top of the oil well is frozen. If it takes 450 N·m of torque to free it and you have a wrench that's 0.40 m long, how much force must you apply?

Answers to Problems about Rotational Motion and Torque

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 17,300 mph

1. Use the equation $v = r\omega$.
2. Convert 1.17×10^{-3} radians/s into radians/hr:

$$\frac{1.17 \times 10^{-3} \text{ radians}}{\text{second}} \times \frac{60 \text{ seconds}}{\text{minute}} \times \frac{60 \text{ minutes}}{\text{hour}} = 4.21 \text{ radians/hr}$$

3. The radius at which the satellite orbits is 150 miles added to the radius of Earth, which is about 3,960 miles, making that radius 4,110 miles. That makes the satellite's tangential speed:

$$v = r\omega = (4,110 \text{ miles})(4.21 \text{ radians/hr}) = 17,300 \text{ mph}$$

2 0.29 radians/s

1. Use the equation $v = r\omega$.
2. Solve for ω :

$$\omega = \frac{v}{r}$$

3. Convert 20.0 mph to ft/hr:

$$\frac{20.0 \text{ miles}}{\text{hour}} \times \frac{5,280 \text{ feet}}{\text{mile}} = 1.06 \times 10^5 \text{ ft/hr}$$

4. Convert from ft/hr to ft/s:

$$\frac{1.06 \times 10^5 \text{ feet}}{\text{hour}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = 29 \text{ ft/s}$$

5. Solve for ω :

$$\omega = \frac{v}{r} = \frac{29 \text{ ft/s}}{100.0 \text{ ft}} = 0.29 \text{ radians/s}$$

3 0.183 radians/s

1. Use the equation $v = r\omega$.
2. Solve for ω :

$$\omega = \frac{v}{r}$$

3. Convert 50.0 mph to ft/hr:

$$\frac{50.0 \text{ miles}}{\text{hour}} \times \frac{5,280 \text{ feet}}{\text{mile}} = 2.64 \times 10^5 \text{ ft/hr}$$

4. Convert from ft/hr to ft/s:

$$\frac{2.64 \times 10^5 \text{ feet}}{\text{hour}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = 73.3 \text{ ft/s}$$

5. Solve for ω :

$$\omega = \frac{v}{r} = \frac{73.3 \text{ ft/s}}{400.0 \text{ ft}} = 0.183 \text{ radians/s}$$

4 240 radians/s

1. Use the equation $v = r\omega$.

2. Solve for ω :

$$\omega = \frac{v}{r}$$

3. Convert 500.0 mph to ft/hr:

$$\frac{500 \text{ miles}}{\text{hour}} \times \frac{5,280 \text{ feet}}{\text{mile}} = 2.6 \times 10^6 \text{ ft/hr}$$

4. Convert from ft/hr to ft/s:

$$\frac{2.6 \times 10^6 \text{ feet}}{\text{hour}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = 730 \text{ ft/s}$$

5. Solve for ω :

$$\omega = \frac{v}{r} = \frac{730 \text{ ft/s}}{3.0 \text{ ft}} = 240 \text{ radians/s}$$

5 0.84 radians / s²

1. Use this equation:

$$a = \frac{\Delta v}{\Delta t} = r\alpha$$

2. Plug in the numbers:

$$a = \frac{\Delta v}{\Delta t} = \frac{88 \text{ m/s} - 0 \text{ m/s}}{(3.5 \text{ min})(60 \text{ s/min})} = r\alpha$$

3. Divide both sides by r :

$$\frac{88 \text{ m/s} - 0 \text{ m/s}}{(0.5 \text{ m})(3.5 \text{ min})(60 \text{ s/min})} = \alpha$$

4. Do the math:

$$\frac{88 \text{ m/s} - 0 \text{ m/s}}{(0.5 \text{ m})(3.5 \text{ min})(60 \text{ s/min})} = \alpha = 0.84 \text{ radians/s}^2$$

6 1.25×10^{-2} radians/s²

1. Use this equation:

$$a = \frac{\Delta v}{\Delta t} = r\alpha$$

2. Plug in the numbers:

$$a = \frac{\Delta v}{\Delta t} = \frac{(30.0 \text{ m/s} - 20.0 \text{ m/s})}{(80.0 \text{ s})} = r\alpha$$

3. Divide both sides by r :

$$\frac{(30.0 \text{ m/s} - 20.0 \text{ m/s})}{(10.0 \text{ m})(80.0 \text{ s})} = \alpha$$

4. Do the math:

$$\frac{(30.0 \text{ m/s} - 20.0 \text{ m/s})}{(10.0 \text{ m})(80.0 \text{ s})} = \alpha = 1.25 \times 10^{-2} \text{ radians/s}^2$$

7 **Downward**

1. Curl your right hand in the direction of rotational motion — clockwise.
2. Your right thumb points downward, indicating the direction of the ω vector.

8 **To the left**

1. Curl your right hand in the direction of rotational motion.
2. Your right thumb points to the left, indicating the direction of the ω vector.

9 **Upward**

1. Curl your right hand in the direction of rotational motion.
2. Your right thumb points downward, indicating the direction of the ω vector.
3. In time, the rotation is slowing, so the ω vector must be decreasing in magnitude. That means α points in the opposite direction — upward.

10 **Downward**

1. Curl your right hand in the direction of rotational motion.
2. Your right thumb points upward, indicating the direction of the ω vector.
3. In time, the rotation is slowing, so the ω vector must be decreasing in magnitude. That means α points in the opposite direction — downward.

11 **130 N·m**

1. Use the equation $\tau = rF \sin \theta$.
2. Plug in the numbers:

$$\tau = rF \sin \theta = (100.0 \text{ N})(1.3 \text{ m}) \sin 90^\circ = 130 \text{ N·m}$$

12 **20 N·m**

1. Use the equation $\tau = rF \sin \theta$.
2. Plug in the numbers:

$$\tau = rF \sin \theta = (200.0 \text{ N})(0.10 \text{ m}) \sin 90^\circ = 20 \text{ N}\cdot\text{m}$$

13 **150 N·m**

1. Use the equation $\tau = rF \sin \theta$.
2. Plug in the numbers:

$$\tau = rF \sin \theta = (200.0 \text{ N})(0.75 \text{ m}) \sin 78^\circ = 147 \text{ N}\cdot\text{m}$$

which rounds to 150 N·m with significant digits

14 **35 N·m**

1. Use the equation $\tau = rF \sin \theta$.
2. Plug in the numbers to give you:

$$\tau = rF \sin \theta = (150 \text{ N})(0.25 \text{ m}) \sin 67^\circ = 35 \text{ N}\cdot\text{m}$$

15 **300 N**

1. Use the equation $\Sigma \tau = 0$.
2. Find the torque you apply using this equation, where L is the width of the door:

$$\tau = rF_{\text{you}} \sin \theta = (100 \text{ N})(L) \sin 90^\circ = 100 L \text{ N}\cdot\text{m}$$

3. Find the torque the other person applies using this equation, where F is the force that person applies:

$$\tau = rF_{\text{other}} \sin \theta = (F)(L/3) \sin 90^\circ = F(L/3) \text{ N}\cdot\text{m}$$

4. To have rotational equilibrium, set these two torques equal and solve for F :

$$\begin{aligned}(100 \text{ N})L &= F(L/3) \\ (3)(100 \text{ N}) &= F = 300 \text{ N}\end{aligned}$$

16 **$2L/3$**

1. Use the equation $\Sigma \tau = 0$.
2. Find the torque the other person applies using this formula, where F is the force that person applies and m is your mass:

$$\tau = rF_{\text{other}} \sin \theta = (2mg)(L/3) \sin 90^\circ = 2mg(L/3) \text{ N}\cdot\text{m}$$

3. Find the torque you apply using this equation, where x is your distance to the pivot:

$$\tau = rF_{\text{you}} \sin \theta = (mg)x \sin 90^\circ = mgx \text{ N}\cdot\text{m}$$

4. To have rotational equilibrium, set these two torques equal and solve for x :

$$\begin{aligned}2mg(L/3)\text{N}\cdot\text{m} &= mgx\text{ N}\cdot\text{m} \\x &= 2L/3\end{aligned}$$

17 **150 m**

1. Use the equation $\Sigma\tau = 0$.
2. Find the torque you apply:

$$\tau = rF \sin \theta = (33\text{ N})x \sin 90^\circ = 33x\text{ N}\cdot\text{m}$$

3. Set that equal to the torque of the motor and solve for x :

$$\begin{aligned}(33\text{ N})x &= 5,000\text{ N}\cdot\text{m} \\x &= \frac{5,000\text{ N}\cdot\text{m}}{33\text{ N}} = 151\text{ m}\end{aligned}$$

which rounds to 150 m with significant figures

18 **1,100 N**

1. Use the equation $\Sigma\tau = 0$.
2. Find the torque you apply:

$$\tau = rF \sin \theta = (F)(0.4\text{ m}) \sin 90^\circ = F(0.4)\text{ N}\cdot\text{m}$$

3. Set that equal to the torque needed and solve for F :

$$\begin{aligned}F(0.4)\text{N}\cdot\text{m} &= 450\text{ N}\cdot\text{m} \\F &= \frac{450\text{ N}\cdot\text{m}}{0.4\text{ N}\cdot\text{m}} = 1,125\text{ N}\end{aligned}$$

which rounds to 1,100 N with significant figures

IN THIS CHAPTER

- » Calculating moments of inertia
- » Doing rotational work
- » Rolling with rotational work and ramps
- » Handling angular momentum
- » Working with rotational dynamics

Chapter 10

Getting Dizzy with Rotational Dynamics

Linear physics translates to rotational physics in several ways — in terms of angular speed and acceleration and in terms of angular momentum and rotational kinetic energy. What's the rotational analog of force? Torque!

How about $F = ma$? What's the analog of that? It turns out to be $\tau = I\alpha$, where I is angular inertia, which is often called the *moment of inertia*. Just about everything in linear motion has an angular analog. This chapter covers the moment of inertia, torque as it relates to angular acceleration, and rotational kinetic energy.

Putting Newton on Wheels

Figure 10-1 shows a ball at the end of stick, moving in a circle. You should remember that the force needed to accelerate the ball around a circle is given by $F = ma$, so the torque, which equals rF , is

$$rF = \tau = mar$$

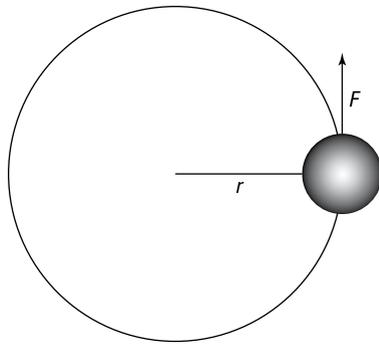


FIGURE 10-1:
A ball moving
in a circle.

© John Wiley & Sons, Inc.

Look at this situation in terms of α , angular acceleration. Angular acceleration is one of those items you can multiply by the radius to get the linear equivalent, which in this case is equal to the tangential acceleration:

$$a = r\alpha$$

Substituting $a = r\alpha$ in the torque equation gives you:

$$\tau = mr^2\alpha$$

This is an important result because it relates torque and angular acceleration. The quantity mr^2 is called the moment of inertia, I , and it represents the effort you need to get something to change its angular velocity, so this equation is usually written as:

$$\tau = I\alpha$$

This equation is a general result, but the moment of inertia, I , differs depending on the situation. For example, I is different when you're spinning a solid cylinder versus a solid sphere. For a single small mass such as the ball on the end of a stick, $I = mr^2$.

The units of moment of inertia are kilogram-meters² (kg·m²) in the MKS system.



EXAMPLE

Q. You're whipping a cannonball around on the end of a 0.50-m steel rod. If the cannonball has a mass of 10.0 kg, what torque do you need to apply to get an angular acceleration of 0.50 radians/s²?

A. The correct answer is 1.25 N·m.

1. Use the equation $\tau = I\alpha = mr^2\alpha$.
2. Plug in the numbers:

$$\begin{aligned}\tau &= mr^2\alpha \\ &= (10.0 \text{ kg})(0.50 \text{ m})^2(0.50 \text{ radians/s}^2) \\ &= 1.25 \text{ N}\cdot\text{m}\end{aligned}$$

The cannonball has so much more mass than the rod that you can ignore the rod's mass entirely.

1 You're rotating a 10.0-kg cannonball at the end of a 1.0-m rod in a circle and want an angular acceleration of 1.0 radian/s^2 . What torque do you need to supply?

2 The cannonball (10 kg) on the end of a 1.0-m rod has an angular acceleration of 2.0 radians/s^2 . What torque are you applying?

3 You're shoveling snow, holding the shovel handle in your right hand. Assuming that you can use the equation $I = mr^2$ to determine the moment of inertia of the shovel, if the 2.0-kg shovel has an angular acceleration of 10.0 radians/s^2 and a length of 1.5 m, what torque are you applying?

4 A 100.0-g clock pendulum on the end of a 1.0-m rod has an angular acceleration of 2.0 radians/s^2 . What torque is being applied?

Moments of Inertia for Everyone

You know that the moment of inertia of a small mass on the end of a thin rod is mr^2 . What are the moments of inertia for other configurations, such as a solid sphere? By treating each mass as a collection of small masses, the moments of inertia for several other shapes have been figured out; here are some of them.



REMEMBER

In the following, m is the total mass of the object, and r is always the radius (of the disk, cylinder, sphere, or hoop). L is the length of the rod, or the length of the rectangle in the direction perpendicular to the axis of rotation.

- » Hoop rotating around its center (like a bicycle tire): $I = mr^2$
- » Solid disk rotating around its center: $I = (1/2)mr^2$
- » Hollow cylinder rotating around its center (such as a car tire): $I = mr^2$
- » Solid cylinder: $I = (1/2)mr^2$
- » Hollow sphere: $I = (2/3)mr^2$
- » Solid sphere: $I = (2/5)mr^2$
- » Point mass rotating at radius r : $I = mr^2$
- » Rod rotating around an axis perpendicular to it and through its center: $I = (1/12)mL^2$
- » Rod rotating around an axis perpendicular to it and through one end: $I = (1/3)mL^2$
- » Rectangle rotating around an axis along one edge: $I = (1/3)mL^2$
- » Rectangle rotating around an axis parallel to one edge and passing through the center: $I = (1/12)mL^2$



EXAMPLE

Q. A solid cylinder with a mass of 5.0 kg is rolling down a ramp. If it has a radius of 10 cm and an angular acceleration of 3.0 radians/s², what torque is operating on it?

A. The correct answer is 0.075 N-m.

1. Use the equation $\tau = I\alpha$.
2. In this case, $I = (1/2)mr^2$
3. Plug in the numbers:

$$\begin{aligned}\tau &= \frac{1}{2}mr^2\alpha \\ &= \frac{1}{2}(5.0 \text{ kg})(0.10 \text{ m})^2(3.0 \text{ radians/s}^2) \\ &= 0.075 \text{ N-m}\end{aligned}$$

5 You're spinning a 5.0-kg ball with a radius of 0.5 m. If it's accelerating at 4.0 radians/s², what torque are you applying?

6 A tire with a radius of 0.50 m and mass of 1.0 kg is rolling down a street. If it's accelerating with an angular acceleration of 10.0 radians/s², what torque is operating on it?

7 You're spinning a hollow sphere with a mass of 10.0 kg and radius of 1.0 m. If it has an angular acceleration of 15 radians/s², what torque are you applying?

8 You're throwing a 300.0-g flying disc with a radius of 10 cm, accelerating it with an angular acceleration of 20.0 radians/s². What torque are you applying?

9 If you're spinning a 2.0-kg solid ball with a radius of 0.5 m, starting from rest and applying a 6.0 N-m torque, what is its angular speed after 60.0 seconds?

10 If you're spinning a 2.0-kg hollow ball with a radius of 0.50 m, starting from rest and applying a 12.0 N-m torque, what is its angular speed after 10.0 seconds?

Doing Some Rotational Work

If you apply a force of 500 N to the edge of a tire to get a car moving, what work do you do over 1.0 m of travel? That work looks like this equation, where s is the distance the force was applied over:

$$W = Fs$$

You can also think about this force rotationally. In the case of you applying force to the edge of a tire to get a car moving, the distance s equals the radius multiplied by the angle through which the wheel turns, $s = r\theta$, so you get this equation:

$$W = Fs = Fr\theta$$

But the torque, τ , equals Fr in this case. So you're left with this:

$$W = Fs = Fr\theta = \tau\theta$$

Talk about a cool result — work equals torque multiplied by the angle through which that torque is applied.



EXAMPLE

Q. If you apply a torque of 500.0 N-m to a tire and turn it through an angle of 2π radians, what work have you done?

A. The correct answer is 3,140 J.

1. Use the equation $W = \tau\theta$.
2. Plug in the numbers:

$$W = \tau\theta = (500.0 \text{ N}\cdot\text{m})(2\pi \text{ radians}) = 3,140 \text{ J}$$

11 How much work do you do if you apply a torque of 6.0 N-m over an angle of 200 radians?

12 You've done 20.0 J of work turning a steering wheel. If you're applying 10.0 N-m of torque, what angle have you turned the steering wheel through?

13 How much work do you do if you apply a torque of 75 N-m through an angle of 6π radians?

14 You've done 350 J of work turning a bicycle tire. If you're applying 150 N-m of torque, what angle have you turned the wheel through?

Round and Round: Rotational Kinetic Energy

In Chapter 7, you review the equation for linear kinetic energy:

$$KE = \frac{1}{2}mv^2$$

Convert that equation to its angular analog:

$$KE = \frac{1}{2}(mr^2)\left(\frac{v}{r}\right)^2 = \frac{1}{2}I\omega^2$$

In other words, the first equation becomes the second when you're going rotational. The angular velocity ω takes the place of the linear velocity v , and the momentum of inertia I takes the place of the mass m .



EXAMPLE

Q. You have a 100-kg solid sphere with a radius equal to 2.0 m. If it's rotating at $\omega = 10.0$ radians/s, what is its rotational kinetic energy?

A. The correct answer is 800 J.

1. Use this equation:

$$KE = \frac{1}{2}I\omega^2$$

2. For a solid sphere, $I = (2/5)mr^2$.

3. Plug in the numbers:

$$\begin{aligned} KE &= \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 \\ &= \left(\frac{1}{2}\right)\left(\frac{2}{5}\right)(100 \text{ kg})(2.0 \text{ m})^2(10.0 \text{ radians/s})^2 \\ &= 800 \text{ J} \end{aligned}$$

15 How much rotational kinetic energy does a spinning tire of mass 10.0 kg and radius 0.50 m have if it's spinning at 40.0 rotations/s?

16 How much rotational kinetic energy does a spinning tire of mass 12 kg and radius 0.80 m have if it's spinning at 200.0 radians/s?

17 How much work do you do to spin a tire, which has a mass of 5.0 kg and a radius of 0.40 m, from 0.0 radians/s to 100.0 radians/s?

18 How much work do you do to spin a hollow sphere, which has a mass of 10.0 kg and a radius of 0.50 m, from 0.0 radians/s to 200.0 radians/s?

Working with Ramps Again

Figure 10-2 shows a hollow cylinder and a solid cylinder at the top of a ramp. What happens when they roll down the ramp? Which cylinder ends up with the greater speed?

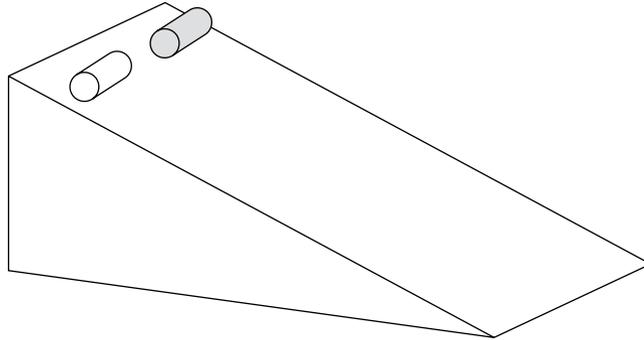


FIGURE 10-2:
A hollow cylinder and a solid cylinder on a ramp.

© John Wiley & Sons, Inc.

You may think that you can simply set the original potential energy equal to the final kinetic energy and solve for the final speed that way:

$$KE = PE = \frac{1}{2}mv^2 = mg\Delta h$$

Unfortunately, you can't work out the answer that way because of what you know about rotational kinetic energy. Some of the potential energy of each cylinder goes into rotational kinetic energy as the cylinders roll to the bottom of the incline. Here's the correct equation to use:

$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Further, you can relate v and ω with the equation $v = r\omega$, which means that $\omega = v/r$, so:

$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v^2}{r^2}\right)$$

Given that equation, this is the final equation for v :

$$v = \sqrt{\frac{2mgh}{m + I/r^2}}$$

So how do you evaluate this for the hollow cylinder and the solid cylinder? For a hollow cylinder, $I = mr^2$; for a solid cylinder, $I = (1/2)mr^2$. Substituting for I for the hollow cylinder gives you this:

$$v = \sqrt{gh}$$

But substituting for I for the solid cylinder gives you this:

$$v = \sqrt{\frac{4gh}{3}}$$

The solid cylinder will be going $\sqrt{\frac{4}{3}} = 1.15$ times as fast as the hollow cylinder when they reach the bottom of the incline.



EXAMPLE

Q. If a solid sphere is at the top of a 3.0-m high ramp, what is its speed when it reaches the bottom of the ramp?

A. The correct answer is 6.5 m/s.

1. Use this equation:

$$v = \sqrt{\frac{2mgh}{m + I/r^2}}$$

2. For a solid sphere, $I = (2/5)mr^2$. That means v is equal to this:

$$v = \sqrt{\frac{2mgh}{m + 2/5m}}$$

3. That breaks down to:

$$v = \sqrt{\frac{2gh}{1 + 2/5}}$$

4. Plug in the numbers:

$$v = \sqrt{\frac{2(9.8 \text{ m/s}^2)(3.0 \text{ m})}{1 + 2/5}} = 6.5 \text{ m/s}$$

19 If a hollow cylinder is at the top of a 4.0-m high ramp, what is its speed when it reaches the bottom of the ramp?

20 If a solid cylinder is at the top of a 2.0-m high ramp, what is its speed when it reaches the bottom of the ramp?

21 A tire is rolling down a ramp, starting at a height of 3.5 m. What is its speed when it reaches the bottom of the ramp?

22 A basketball (that is, a hollow sphere) is rolling down a ramp, starting at a height of 4.8 m. What is its speed when it reaches the bottom of the ramp?

Can't Stop This: Angular Momentum

In linear motion, momentum looks like this:

$$\mathbf{p} = m\mathbf{v}$$

Momentum is conserved in collisions. In addition to linear momentum, you can have angular momentum, which is represented by the symbol \mathbf{L} . Following is the equation for angular momentum:

$$\mathbf{L} = I\omega$$

Note that \mathbf{L} , like ω , is actually a vector. In the simple cases, consider that \mathbf{L} always points in the same direction as ω .

In physics, angular momentum is conserved. For example, if you are a skater spinning around and then you spread your arms (giving you a different moment of inertia), because angular momentum is conserved, this is the equation:

$$I_1\omega_1 = I_2\omega_2$$

With this equation, if you, as the skater, know your original and final moments of inertia and your original angular speed, you can calculate your final angular speed like this:

$$\omega_2 = \frac{I_1}{I_2}\omega_1$$



EXAMPLE

Q. If a 500.0-kg merry-go-round with a radius of 2.0 m is spinning at 2.0 radians/s and a boy with a mass of 40.0 kg jumps on the outer rim, what is the new angular speed of the merry-go-round?

A. The correct answer is 1.7 radians/s.

1. Use this equation:

$$I_1\omega_1 = I_2\omega_2$$

2. For a solid disc like the merry-go-round, $I = (1/2)mr^2$.

3. When the boy jumps on, he adds $m_b r^2$ to I , where m_b is the mass of the boy. This means that:

$$\begin{aligned} \left(\frac{1}{2}mr^2\right)\omega_1 &= \left(\frac{1}{2}mr^2 + m_b r^2\right)\omega_2 \\ 2,000 \text{ kg}\cdot\text{m}^2\text{-radians/s} &= \left(\frac{1}{2}mr^2 + m_b r^2\right)\omega_2 \end{aligned}$$

4. Solve for ω_2 :

$$\omega_2 = \frac{\frac{1}{2}mr^2\omega_1}{\frac{1}{2}mr^2 + m_b r^2}$$

where $(1/2)mr^2 = 1,000 \text{ kg}\cdot\text{m}^2$ and $m_b r^2 = 160 \text{ kg}\cdot\text{m}^2$.

5. Plug in the numbers:

$$\omega_2 = \frac{2,000 \text{ kg}\cdot\text{m}^2}{1,000 \text{ kg}\cdot\text{m}^2 + 160 \text{ kg}\cdot\text{m}^2} = 1.7 \text{ radians/s}$$

23 A merry-go-round with a mass of 500.0 kg and radius of 2.0 m is rotating at 3.0 radians/s when two children with a combined mass of 70.0 kg jump on the outer rim. What is the new angular speed of the merry-go-round?

24 A 2,000.0-kg space station, which is a hollow cylinder with a radius of 2.0 m, is rotating at 1.0 radian/s when an astronaut with a mass of 80.0 kg lands on the outside of the station. What is the station's new angular speed?

Answers to Problems about Rotational Dynamics

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 10 N-m

1. Use the equation $\tau = mr^2\alpha$.
2. Plug in the numbers:

$$\tau = mr^2\alpha = (10.0 \text{ kg})(1.0 \text{ m})^2(1.0 \text{ radian/s}^2) = 10 \text{ N-m}$$

2 20 N-m

1. Use the equation $\tau = mr^2\alpha$.
2. Plug in the numbers:

$$\tau = mr^2\alpha = (10 \text{ kg})(1.0 \text{ m})^2(2.0 \text{ radians/s}^2) = 20 \text{ N-m}$$

3 45 N-m

1. Use the equation $\tau = mr^2\alpha$.
2. Plug in the numbers:

$$\tau = mr^2\alpha = (2.0 \text{ kg})(1.5 \text{ m})^2(10.0 \text{ radians/s}^2) = 45 \text{ N-m}$$

4 0.20 N-m

1. Use the equation $\tau = mr^2\alpha$.
2. Plug in the numbers:

$$\tau = mr^2\alpha = (0.1 \text{ kg})(1.0 \text{ m})^2(2.0 \text{ radians/s}^2) = 0.20 \text{ N-m}$$

5 2 N-m

1. Use the equation $\tau = I\alpha$.
2. In this case, $I = (2/5)mr^2$.
3. Plug in the numbers:

$$\tau = \frac{2}{5}mr^2\alpha = \frac{2}{5}(5.0 \text{ kg})(0.5 \text{ m})^2(4.0 \text{ radians/s}^2) = 2 \text{ N-m}$$

6 2.5 N-m

1. Use the equation $\tau = I\alpha$.
2. In this case, $I = mr^2$.
3. Plug in the numbers:

$$\tau = mr^2\alpha = (1.0 \text{ kg})(0.50 \text{ m})^2(10.0 \text{ radians/s}^2) = 2.5 \text{ N-m}$$

7 100 N·m

1. Use the equation $\tau = I\alpha$.
2. In this case, $I = (2/3)mr^2$.
3. Plug in the numbers:

$$\tau = \frac{2}{3}mr^2\alpha = \frac{2}{3}(10.0 \text{ kg})(1.0 \text{ m})^2(15 \text{ radians/s}^2) = 100 \text{ N·m}$$

8 0.03 N·m

1. Use the equation $\tau = I\alpha$.
2. In this case, $I = (1/2)mr^2$.
3. Plug in the numbers:

$$\tau = \frac{1}{2}mr^2\alpha = \frac{1}{2}(0.3 \text{ kg})(0.10 \text{ m})^2(20.0 \text{ radians/s}^2) = 0.03 \text{ N·m}$$

9 1,800 radians/s

1. Use the equation $\tau = I\alpha$.
2. In this case, $I = (2/5)mr^2$.
3. Solve for α and plug in the numbers:

$$\alpha = \frac{\tau}{I} = \frac{6.0 \text{ N·m}}{\left(\frac{2}{5}\right)(2.0 \text{ kg})(0.5 \text{ m})^2} = 30.0 \text{ radians/s}^2$$

4. Use the equation $\omega = \alpha t$ and plug in the numbers:

$$\omega = \alpha t = (30.0 \text{ radians/s}^2)(60 \text{ s}) = 1,800 \text{ radians/s}$$

10 360 radians/s

1. Use the equation $\tau = I\alpha$.
2. In this case, $I = (2/3)mr^2$.
3. Solve for α and plug in the numbers:

$$\alpha = \frac{\tau}{I} = \frac{12.0 \text{ N·m}}{\left(\frac{2}{3}\right)(2.0 \text{ kg})(0.50 \text{ m})^2} = 36 \text{ radians/s}^2$$

4. Use the equation $\omega = \alpha t$ and plug in the numbers:

$$\omega = \alpha t = (36 \text{ radians/s}^2)(10.0 \text{ s}) = 360 \text{ radians/s}$$

11 1,200 J

1. Use the equation $W = \tau\theta$.
2. Plug in the numbers:

$$W = \tau\theta = (6.0 \text{ N·m})(200 \text{ radians}) = 1,200 \text{ J}$$

12 2 radians

1. Use the equation $W = \tau\theta$.
2. Solve for θ and plug in the numbers:

$$\theta = \frac{W}{\tau} = \frac{20.0 \text{ J}}{10.0 \text{ N}\cdot\text{m}} = 2 \text{ radians}$$

13 1,400 J

1. Use the equation $W = \tau\theta$.
2. Plug in the numbers:

$$W = \tau\theta = (75 \text{ N}\cdot\text{m})(6\pi) = 1,400 \text{ J}$$

14 2.3 radians

1. Use the equation $W = \tau\theta$.
2. Solve for θ and plug in the numbers:

$$\theta = \frac{W}{\tau} = \frac{350 \text{ J}}{150 \text{ N}\cdot\text{m}} = 2.3 \text{ radians}$$

15 $7.9 \times 10^4 \text{ J}$

1. Use the equation for kinetic energy:

$$KE = \frac{1}{2}I\omega^2$$

2. For a spinning tire, $I = mr^2$.
3. Convert 40 rotations/s to radians/s. One rotation is 2π radians, so 40 rotations/s is $40(2\pi) = 80\pi$ radians/s.
4. Plug in the numbers:

$$KE = \frac{1}{2}I\omega^2 = \frac{1}{2}(10.0 \text{ kg})(0.50 \text{ m})^2(80\pi)^2 = 7.9 \times 10^4 \text{ J}$$

16 $1.5 \times 10^5 \text{ J}$

1. Use the equation for kinetic energy:

$$KE = \frac{1}{2}I\omega^2$$

2. For a spinning tire, $I = mr^2$.
3. Plug in the numbers:

$$KE = \frac{1}{2}I\omega^2 = \frac{1}{2}(12 \text{ kg})(0.80 \text{ m})^2(200.0 \text{ radians/s})^2 = 1.5 \times 10^5 \text{ J}$$

17 **4,000 J**

1. The work you do goes into the tire's final kinetic energy, so use the equation for kinetic energy:

$$KE = \frac{1}{2} I \omega^2$$

2. For a spinning tire, $I = mr^2$.
3. Plug in the numbers:

$$KE = \frac{1}{2} I \omega^2 = \frac{1}{2} (5.0 \text{ kg})(0.40 \text{ m})^2 (100.0 \text{ radians/s})^2 = 4,000 \text{ J}$$

18 **33,000 J**

1. The work you do goes into the sphere's final kinetic energy, so use the equation for kinetic energy:

$$KE = \frac{1}{2} I \omega^2$$

2. For a hollow sphere, $I = (2/3)mr^2$.
3. Plug in the numbers:

$$KE = \frac{1}{2} I \omega^2 = \frac{1}{2} \left(\frac{2}{3} \right) (10.0 \text{ kg})(0.50 \text{ m})^2 (200.0 \text{ radians/s})^2 = 33,300 \text{ J}$$

which rounds to 33,000 with significant figures

19 **6.3 m/s**

1. Use this equation:

$$v = \sqrt{\frac{2mgh}{m + I/r^2}}$$

2. For a hollow cylinder, $I = mr^2$.
3. That means v is equal to this:

$$v = \sqrt{\frac{2mgh}{m + m}}$$

4. This equation breaks down to:

$$v = \sqrt{gh}$$

5. Plug in the numbers:

$$v = \sqrt{(9.8 \text{ m/s}^2)(4.0 \text{ m})} = 6.3 \text{ m/s}$$

20 **5.1 m/s**

1. Use this equation:

$$v = \sqrt{\frac{2mgh}{m + I/r^2}}$$

2. For a solid cylinder, $I = (1/2)mr^2$.

3. That means v is equal to this:

$$v = \sqrt{\frac{2mgh}{m + (1/2)m}}$$

4. This equation breaks down to:

$$v = \sqrt{\frac{2gh}{1 + 1/2}}$$

5. Plug in the numbers:

$$v = \sqrt{\frac{2(9.8 \text{ m/s}^2)(2.0 \text{ m})}{1 + 1/2}} = 5.1 \text{ m/s}$$

21 **5.9 m/s**

1. Use this equation:

$$v = \sqrt{\frac{2mgh}{m + I/r^2}}$$

2. For a tire, $I = mr^2$.

3. That means v is equal to this:

$$v = \sqrt{\frac{2mgh}{m + m}}$$

4. This equation breaks down to:

$$v = \sqrt{gh}$$

5. Plug in the numbers:

$$v = \sqrt{(9.8 \text{ m/s}^2)(3.5 \text{ m})} = 5.9 \text{ m/s}$$

22 **7.5 m/s**

1. Use this equation:

$$v = \sqrt{\frac{2mgh}{m + I/r^2}}$$

2. For a hollow sphere, $I = (2/3)mr^2$.

3. That means v is equal to this:

$$v = \sqrt{\frac{2mgh}{m + (2/3)m}}$$

4. This equation breaks down to:

$$v = \sqrt{\frac{2gh}{1 + 2/3}}$$

5. Plug in the numbers:

$$v = \sqrt{\frac{2(9.8 \text{ m/s}^2)(4.8 \text{ m})}{1 + 2/3}} = 7.5 \text{ m/s}$$

23 2.3 radians/s

1. Use this equation:

$$I_1\omega_1 = I_2\omega_2$$

2. For a solid disc like the merry-go-round, $I = (1/2)mr^2$.

3. When the children jump on, that adds $m_c r^2$ to I , where m_c is the mass of the children. In other words:

$$\frac{1}{2}mr^2\omega_1 = \left(\frac{1}{2}mr^2 + m_c r^2\right)\omega_2$$

4. Solve for ω_2 to get

$$\omega_2 = \frac{\frac{1}{2}mr^2\omega_1}{\frac{1}{2}mr^2 + m_c r^2}$$

5. Plug in the numbers:

$$\omega_2 = \frac{3,000 \text{ kg}\cdot\text{m}^2\text{-radians/s}}{1,000 \text{ kg}\cdot\text{m}^2 + 280 \text{ kg}\cdot\text{m}^2} = 2.3 \text{ radians/s}$$

24 0.96 radians/s

1. Use this equation:

$$I_1\omega_1 = I_2\omega_2$$

2. For a hollow cylinder like the space station, $I = mr^2$.

3. When the astronaut lands, that adds $m_a r^2$ to I , where m_a is the mass of the astronaut. That change means the following equation applies:

$$mr^2\omega_1 = (mr^2 + m_a r^2)\omega_2$$

4. Solve for ω_2 :

$$\omega_2 = \frac{mr^2\omega_1}{mr^2 + m_a r^2}$$

5. Plug in the numbers:

$$\omega_2 = \frac{8,000 \text{ kg}\cdot\text{m}^2\text{-radians/s}}{8,000 \text{ kg}\cdot\text{m}^2 + 320 \text{ kg}\cdot\text{m}^2} = 0.96 \text{ radians/s}$$

IN THIS CHAPTER

- » Using Hooke's law
- » Following simple harmonic motion
- » Calculating simple harmonic motion velocity
- » Finding simple harmonic motion acceleration
- » Calculating a spring's period of oscillation

Chapter 11

Simple Harmonic Motion

Simple harmonic motion — the motion of springs — is a very important topic in physics. This kind of motion is all based on Hooke's law, which says that the force on a spring in simple harmonic motion is proportional to the distance the object is away from its equilibrium position (the location where the oscillator will feel no unbalanced force). In other words, the farther a spring is stretched, the more it pulls back.

Hooking into Hooke's Law

Hooke's law is represented by this equation:

$$F = -k\Delta x$$



REMEMBER

This deceptively simple equation is at the heart of explaining the motion of objects on springs. It says that the force on an object in simple harmonic motion is proportional to the displacement (that's Δx) from rest.

The constant of proportionality is k (Hooke's constant, also called a spring constant), which must be measured for every situation (because no two springs are exactly identical, for example). The negative sign in the equation indicates that k is a restoring force — that is, that the force points toward the equilibrium position of the object.

What are the units of spring constants? Just check out the equation: Because $F = -k\Delta x$, k must have the unit N/m.

You'll commonly see Hooke's law applied to springs. Don't get confused by the minus sign in this equation; it's just there to indicate that the force opposes the displacement, which you know is true about springs.



TIP

If it's easier for you to understand, put in the minus sign after you've done the rest of the problem. You can easily figure out which way the force is going, and if it's in a positive direction as defined by the axes in the problem, the force is positive.



Q. You're stretching a spring with spring constant 5.0 N/m. If you stretch it +2.0 m, what pull do you feel from the spring?

EXAMPLE

A. The correct answer is -10.0 N.

1. Use the equation $F = -k\Delta x$.

2. Plug in the numbers:

$$F = -k\Delta x = -(5.0 \text{ N/m})(2.0 \text{ m}) = -10.0 \text{ N}$$

1 You have a spring whose spring constant is 200 N/m, and you want to stretch it by 6.0 m. What force do you need to apply?

2 You have a spring of spring constant 50.0 N/m. What force do you need to stretch it by 5.3 m?

3 You have a spring of spring constant 73 N/m. How much does the spring stretch if you apply a force of 52 N?

4 It takes 200.0 N to stretch a spring 2.0 m. What is its spring constant?

Simply Simple Harmonic Motion

Take a look at the spring in Figure 11-1. It starts at rest, drops down to position $-A$, and then moves back up to position A .

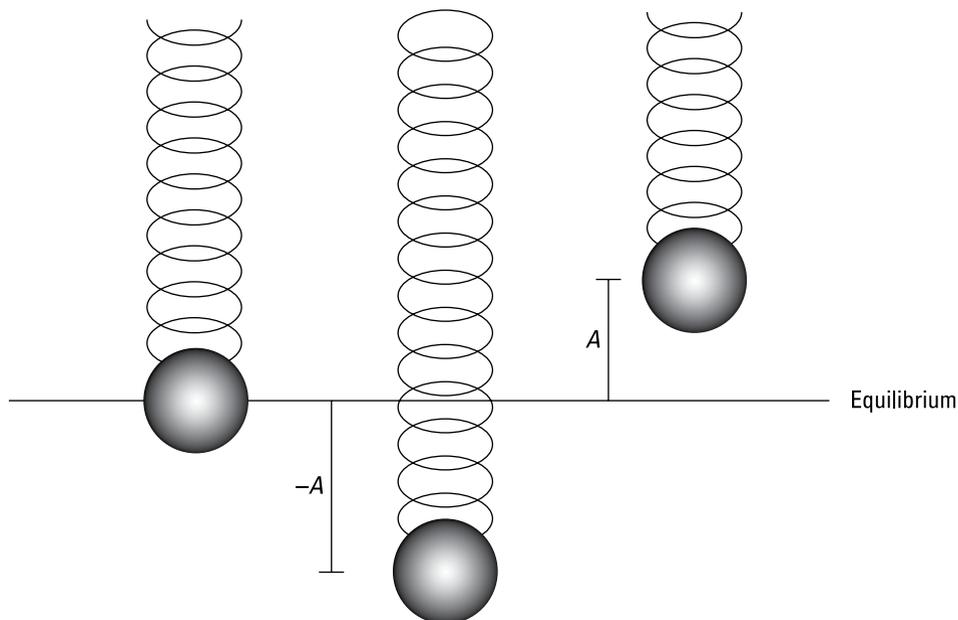


FIGURE 11-1:
A weight on a
spring.

© John Wiley & Sons, Inc.

How do you describe this motion? In terms of position A , called the *amplitude* of the spring's motion:

$$y = A \cos (\omega t)$$

In this equation for displacement, t is time, and ω is the *angular speed*. (It's called angular speed here for a variety of reasons; it turns out that simple harmonic motion is only one component — hence the cosine — of full circular motion.)

That equation assumes that you start at full extension, at time $t = 0$. If you start at a different time, t_o , you can adjust the equation to match the spring's motion like this:

$$y = A \cos [\omega(t - t_o)]$$



EXAMPLE

Q. A weight on a spring is making that spring oscillate up and down. If the amplitude of the motion is 1.0 m, the angular speed is 1.0 radians/s, and the weight is initially at its maximum upward position, where will the oscillating weight be after 10.0 seconds?

A. The correct answer is -0.84 m.

1. Use the equation $y = A \cos(\omega t)$.

2. Plug in the numbers:

$$y = A \cos(\omega t) = (1.0 \text{ m}) \cos[(1.0 \text{ radians/s})(10.0 \text{ s})] = -0.84 \text{ m}$$

The negative sign indicates that the spring is extended downward.



TIP

If you plug the numbers in this equation into your calculator, either put it into radian mode because ωt is in radians or convert to degrees by multiplying ωt by $180 / \pi$.

5 A spring with a weight on it has an amplitude of motion of 2.5 m and an angular speed of 2.0 radians/s. If the weight starts at maximum extension, where will it be after 60.0 seconds?

6 A spring with a weight pulling it down has an amplitude of motion of 5.0 m and an angular speed of 16.0 radians/s. If the weight starts at maximum extension, where will it be after 60.0 seconds?

7 A spring starts at maximum extension, is at -3.0 m at time 60.0 s, and has an angular speed of 6.0 radians/s. What is its amplitude?

8 A spring starts at maximum extension, is at -4.5 m at time 10.0 s, and has an angular speed of 16.0 radians/s. What is its amplitude?

Getting Periodic

A weight on the end of a spring bounces up and down periodically. The time it takes to bounce up and down, completing a full cycle and coming back to where it started, is called its *period*, represented by symbol T . Because $y = A \cos(\omega t)$, the object goes through 2π radians in period T , so you have this relation:

$$\omega = \frac{2\pi}{T}$$

In other words:

$$T = \frac{2\pi}{\omega}$$

The period, T , is measured in seconds.

Besides the period, oscillations are measured in *frequency*, which is the number of cycles per second. The frequency of a simple harmonic oscillator is equal to the inverse of the period. The equation for frequency is

$$f = \frac{1}{T}$$

Because $\omega = 2\pi / T$, you can change the frequency equation to get this equation:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Frequency is measured in Hertz, abbreviation Hz, which is one cycle per second.



TECHNICAL
STUFF

It's worth noting that you can call ω angular speed, but when you're working with simple harmonic motion, it's usually called angular frequency.



EXAMPLE

Q. A weight on a spring is bouncing up and down with an angular frequency of 12.0 radians/s. How long does it take to complete each cycle?

A. The correct answer is 0.52 s.

1. Use this equation:

$$T = \frac{2\pi}{\omega}$$

2. Plug in the numbers:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{12.0 \text{ radians/s}} = 0.52 \text{ s}$$

9 A spring has a weight with an angular frequency of 4.5 radians/s. What is the weight's period?

10 A spring has a weight pulling it down, and the angular frequency of oscillation is 1.5 radians/s. What is the weight's period?

11 A spring with a weight on it has an angular frequency of 0.70 radians/s. What is its frequency?

12 A spring with a weight on it is bouncing up and down with an angular frequency of 1.3 radians/s. What is its frequency?

Considering Velocity

As discussed in the earlier section “Simply Simple Harmonic Motion,” the displacement of an object on a spring looks like this:

$$y = A \cos(\omega t)$$

It turns out that you also can express the object's velocity; here's that equation:

$$v_y = -A\omega \sin \theta = -A\omega \sin (\omega t)$$



TIP

This equation assumes that you start at full extension, at time $t = 0$, which means the initial velocity of the object is zero. If you want to adjust this equation to start at some other time, t_o , you alter the equation like so:

$$v_y = -A\omega \sin [\omega(t - t_o)]$$

When the object is at its fullest extension, the velocity is zero, and when it's swooping back through the equilibrium point, its velocity is at its maximum.



EXAMPLE

Q. A weight on a spring is bouncing up and down with an angular frequency of 3.4 radians/s and an amplitude of 1.4 m. If the weight starts at maximum displacement at $t = 0$, what is its speed at $t = 5.0$ s?

A. The correct answer is 4.6 m/s.

1. Use this equation:

$$v_y = -A\omega \sin (\omega t)$$

2. Plug in the numbers:

$$\begin{aligned} v_y &= -A\omega \sin (\omega t) \\ &= -(1.4 \text{ m})(3.4 \text{ radians/s}) \sin [(3.4 \text{ radians/s})(5.0 \text{ s})] \\ &= 4.6 \text{ m/s} \end{aligned}$$

13 If you have a spring with a weight on it, and the weight's angular frequency is 1.7 radians/s and amplitude is 5.6 m, what is the weight's velocity at 30.0 s, assuming the spring starts at full extension?

14 A spring with a weight on it is bouncing up and down with an angular frequency of 2.7 radians/s and amplitude of 1.7 m. What is its velocity at 10.0 s after it reaches maximum displacement?

Figuring the Acceleration

In addition to calculating the displacement and velocity of an object on a spring, you can calculate its acceleration.

Displacement goes from $-A$ to A , where A is the amplitude. And the velocity of an object on a spring goes from $-A\omega$ to $A\omega$; the speed is at its minimum (zero) at either end of the spring's maximum extension, and it's at a maximum at the equilibrium point (where the object would be at rest). The acceleration varies from $-A\omega^2$ to $A\omega^2$, and it's at its maximum when the object is speeding up (or down!) the most, which is at the ends of its oscillations (when the velocity is zero). The acceleration of the object is zero when the object is passing through the equilibrium position because the net force on the object is zero at the equilibrium position.



REMEMBER

Here's the equation for the acceleration of an object on a spring:

$$a = -A\omega^2 \cos \theta = -A\omega^2 \cos(\omega t)$$

This equation assumes that you start at full extension at $t = 0$. If your object reaches full extension at a different time $t = t_0$, you can use this equation instead:

$$a = -A\omega^2 \cos[\omega(t - t_0)]$$



EXAMPLE

Q. A weight on a spring is bouncing up and down with an angular frequency of 3.4 radians/s and an amplitude of 1.4 m. If the weight starts at maximum displacement, what is its acceleration at 5.0 s?

A. The correct answer is 4.5 m/s².

1. Use this equation:

$$a = -A\omega^2 \cos(\omega t)$$

2. Plug in the numbers:

$$\begin{aligned} a &= -A\omega^2 \cos(\omega t) \\ &= -(1.4 \text{ m})(3.4 \text{ radians/s})^2 \cos[(3.4 \text{ radians/s})(5.0 \text{ s})] \\ &= 4.5 \text{ m/s}^2 \end{aligned}$$

15 A spring has a weight on it and starts at full extension. If the weight's angular frequency is 1.3 radians/s and its amplitude is 1.0 m, what is the weight's acceleration at $t = 4.9$ s?

16 You have a spring with a weight pulling it down; the angular frequency is 1.7 radians/s and the amplitude is 6.0 m. What is the weight's acceleration 60 s after it reaches full extension?

17 A spring with a weight on it has an angular frequency of 3.7 radians/s and an amplitude of 1.4 m. The weight begins at maximum displacement. What is the weight's acceleration at 15 s?

18 A spring with a weight on it is moving up and down with an angular frequency of 1.3 radians/s and an amplitude of 2.9 m. What is its acceleration at 8.0 s if the weight is at maximum displacement at 1.0 s?

Bouncing Around with Springs

What is the period of a spring in terms of its spring constant, k ? You'll often come across that question in physics problems. You know that:

$$F = -k\Delta x$$

You also know that $F = ma$, so:

$$F = ma = -k\Delta x$$

For simple harmonic motion, you know that:

$$x = A \cos(\omega t)$$
$$a = -A\omega^2 \cos(\omega t)$$

Putting this all together gives you:

$$ma = -mA\omega^2 \cos(\omega t) = -k\Delta x = -kA \cos(\omega t)$$

That lengthy equation becomes

$$m\omega^2 = k$$

Which, if you solve for ω , gives you the angular frequency of an object on a spring:

$$\omega = \sqrt{\frac{k}{m}}$$



EXAMPLE

Q. A weight on a spring is bouncing up and down. The spring constant is 1.6 N/m, the mass is 1.0 kg, and the amplitude is 3.0 m. What equation describes the spring's motion?

A. The correct answer is $y = A \cos(\omega t) = (3.0 \text{ m}) \cos(1.3 \text{ radians/s})t$.

1. Use this equation:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1.6 \text{ N/m}}{1.0 \text{ kg}}} = 1.3 \text{ radians/s}$$

2. So this equation describes the motion:

$$y = A \cos(\omega t) = (3.0 \text{ m}) \cos(1.3 \text{ radians/s})t$$

19 If you have a spring with a mass of 1.0 kg on it and a spring constant of 12.0 N/m, what is the spring's period of oscillation?

20 What is the period of oscillation of a spring with a mass of 300 g on it and a spring constant of 7.0 N/m?

- 21 A weight on a spring is moving up and down. The spring constant is 1.9 N/m, the mass is 1.0 kg, and the amplitude is 2.4 m. The weight starts at maximum displacement. What equation describes the spring's motion?

- 22 A weight on a spring is oscillating up and down. The spring constant is 2.3 N/m, the mass is 1.3 kg, and the amplitude is 5.6 m. What equation describes the spring's motion if the spring starts at full extension?

Talking about Energy

When you stretch a spring, you transfer potential energy to that spring (just like when you lift a weight against the force of gravity). For a spring of spring constant k stretched a distance x from equilibrium, the potential energy in the spring is

$$PE = \frac{1}{2}kx^2$$



- Q.** What is the potential energy in a spring of spring constant 40 N/m that's stretched 5.0 m from equilibrium?

EXAMPLE

- A.** The correct answer is 500 J.

1. Use this equation:

$$PE = \frac{1}{2}kx^2$$

2. Plug in the numbers:

$$\begin{aligned} PE &= \frac{1}{2}kx^2 \\ &= \frac{1}{2}(40 \text{ N/m})(5.0 \text{ m})^2 \\ &= 500 \text{ J} \end{aligned}$$

23 If you stretch a spring of spring constant 100 N/m by 5.0 m, what potential energy is in the spring?

24 If you stretch a spring of spring constant 250 N/m by 6.5 m, what potential energy is in the spring?

Following the Ticktock of Pendulums

In addition to springs, physics problems about simple harmonic motion may ask you to deal with pendulums like the one in Figure 11-2.

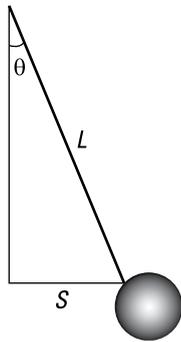


FIGURE 11-2:
A pendulum.

© John Wiley & Sons, Inc.

You can calculate the angular frequency of a pendulum of length L using the following equation, where g is the acceleration due to gravity:

$$\omega = \sqrt{\frac{g}{L}}$$



EXAMPLE

Q. What's the angular frequency of a pendulum of length 1.0 m?

A. The correct answer is 3.1 radians/s.

Use this equation and plug in the numbers:

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8 \text{ m/s}^2}{1.0 \text{ m}}} = 3.1 \text{ radians/s}$$

25 What's the period of a pendulum with a length of 1.5 m?

26 What's the period of a pendulum with a length of 3.0 m?

27 If you have a pendulum with a length of 1.0 m with a weight on the end of it, what equation describes the x position of that weight if the amplitude of motion is 0.5 m?

28 You have a pendulum with a length of 1.5 m with a weight on the end of it. What equation describes the x position of that weight if the amplitude of motion is 0.75 m?

Answers to Problems about Simple Harmonic Motion

The following are the answers to the practice questions presented in this chapter. You see how to work out each answer, step by step.

1 **1,200 N**

1. Use the equation $F = k\Delta x$.
2. Plug in the numbers:

$$F = k\Delta x = (200 \text{ N/m})(6.0 \text{ m}) = 1,200 \text{ N}$$

Note that the sign is positive here because you're pulling on the spring; you're not interested in the force with which the spring pulls back ($-1,200 \text{ N}$).

2 **265 N**

1. Use the equation $F = k\Delta x$.
2. Plug in the numbers:

$$F = k\Delta x = (50.0 \text{ N/m})(5.3 \text{ m}) = 265 \text{ N}$$

Note that the sign is positive here because you're pulling on the spring; you're not interested in the force with which the spring pulls back.

3 **0.71 m**

1. Use the equation $F = k\Delta x$.
2. Solve for Δx :

$$\Delta x = F / k$$

3. Plug in the numbers:

$$\Delta x = F / k = (52 \text{ N}) / (73 \text{ N/m}) = 0.71 \text{ m}$$

4 **100 N/m**

1. Use the equation $F = k\Delta x$.
2. Solve for k :

$$k = F / \Delta x$$

3. Plug in the numbers:

$$k = F / \Delta x = (200.0 \text{ N}) / (2.0 \text{ m}) = 100 \text{ N/m}$$

5 **2.04 m**

1. Use the equation $y = A \cos(\omega t)$.
2. Plug in the numbers:

$$y = A \cos(\omega t) = (2.5 \text{ m}) \cos[(2.0 \text{ radians/s})(60.0 \text{ s})] = 2.04 \text{ m}$$

6 **1.2 m**

1. Use the equation $y = A \cos(\omega t)$.
2. Plug in the numbers:

$$y = A \cos(\omega t) = (5.0 \text{ m}) \cos[(16.0 \text{ radians/s})(60.0 \text{ s})] = 1.2 \text{ m}$$

7 **10.6 m**

1. Use the equation $y = A \cos(\omega t)$.
2. Solve for A and plug in the numbers:

$$A = y / \cos(\omega t) = -3.0 \text{ m} / \cos[(6.0 \text{ radians/s})(60.0 \text{ s})] = 10.6 \text{ m}$$

8 **4.6 m**

1. Use the equation $y = A \cos(\omega t)$.
2. Solve for A and plug in the numbers:

$$A = y / \cos(\omega t) = -4.5 \text{ m} / \cos[(16.0 \text{ radians/s})(10.0 \text{ s})] = 4.6 \text{ m}$$

9 **1.4 s**

1. Use this equation:

$$T = \frac{2\pi}{\omega}$$

2. Plug in the numbers:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4.5 \text{ radians/s}} = 1.4 \text{ s}$$

10 **4.2 s**

1. Use this equation:

$$T = \frac{2\pi}{\omega}$$

2. Plug in the numbers:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.5 \text{ radians/s}} = 4.2 \text{ s}$$

11 **0.11 cycles/s**

1. Use this equation:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

2. Solve for f :

$$f = \frac{\omega}{2\pi}$$

3. Plug in the numbers:

$$f = \frac{\omega}{2\pi} = \frac{0.70 \text{ radians/s}}{2\pi} = 0.11 \text{ cycles/s}$$

12 **0.21 cycles/s**

1. Use this equation:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

2. Solve for f :

$$f = \frac{\omega}{2\pi}$$

3. Plug in the numbers:

$$f = \frac{\omega}{2\pi} = \frac{1.3 \text{ radians/s}}{2\pi} = 0.21 \text{ cycles/s}$$

13 **-6.38 m/s**

1. Use this equation:

$$v_y = -A\omega \sin(\omega t)$$

2. Plug in the numbers:

$$\begin{aligned} v_y &= -A\omega \sin(\omega t) \\ &= -(5.6 \text{ m})(1.7 \text{ radians/s}) \sin [(1.7 \text{ radians/s})(30.0 \text{ s})] \\ &= -6.38 \text{ m/s} \end{aligned}$$

14 **-4.4 m/s**

1. Use this equation:

$$v_y = -A\omega \sin(\omega t)$$

2. Plug in the numbers:

$$\begin{aligned} v_y &= -A\omega \sin(\omega t) \\ &= -(1.7 \text{ m})(2.7 \text{ radians/s}) \sin [(2.7 \text{ radians/s})(10.0 \text{ s})] \\ &= -4.4 \text{ m/s} \end{aligned}$$

15 **-1.7 m/s²**

1. Use this equation:

$$a = -A\omega^2 \cos(\omega t)$$

2. Plug in the numbers:

$$\begin{aligned}a &= -A\omega^2 \cos(\omega t) \\ &= -(1.0 \text{ m})(1.3 \text{ radians/s})^2 \cos[(1.3 \text{ radians/s})(4.9 \text{ s})] \\ &= -1.7 \text{ m/s}^2\end{aligned}$$

16 **-1.8 m/s²**

1. Use this equation:

$$a = -A\omega^2 \cos(\omega t)$$

2. Plug in the numbers:

$$\begin{aligned}a &= -A\omega^2 \cos(\omega t) \\ &= -(6.0 \text{ m})(1.7 \text{ radians/s})^2 \cos[(1.7 \text{ radians/s})(60 \text{ s})] \\ &= -1.8 \text{ m/s}^2\end{aligned}$$

17 **-10 m/s²**

1. Use this equation:

$$a = -A\omega^2 \cos(\omega t)$$

2. Plug in the numbers:

$$\begin{aligned}a &= -A\omega^2 \cos(\omega t) \\ &= -(1.4 \text{ m})(3.7 \text{ radians/s})^2 \cos[(3.7 \text{ radians/s})(15 \text{ s})] \\ &= -10 \text{ m/s}^2\end{aligned}$$

18 **4.6 m/s²**

1. Use this equation:

$$a = -A\omega^2 \cos[\omega(t - t_0)]$$

2. Plug in the numbers:

$$\begin{aligned}a &= -A\omega^2 \cos[\omega(t - t_0)] \\ &= -(2.9 \text{ m})(1.3 \text{ radians/s})^2 \cos[(1.3 \text{ radians/s})(8.0 \text{ s} - 1.0 \text{ s})] \\ &= 4.6 \text{ m/s}^2\end{aligned}$$

19 **1.8 s**

1. Use this equation and plug in the numbers:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{12.0 \text{ N/m}}{1.0 \text{ kg}}} = 3.5 \text{ radians/s}$$

2. Now use this equation and plug in the numbers:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{3.5 \text{ radians/s}} = 1.8 \text{ s}$$

20 **1.3 s**

1. Use this equation:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{7.0 \text{ N/m}}{0.3 \text{ kg}}} = 4.8 \text{ radians/s}$$

2. Now use this equation:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4.8 \text{ radians/s}} = 1.3 \text{ s}$$

21 **$y = A \cos(\omega t) = (2.4 \text{ m}) \cos[(1.4 \text{ radians/s})t]$**

1. Use this equation:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1.9 \text{ N/m}}{1.0 \text{ kg}}} = 1.4 \text{ radians/s}$$

2. Now use this equation:

$$y = A \cos(\omega t) = (2.4 \text{ m}) \cos(1.4 \text{ radians/s})t$$

22 **$y = A \cos(\omega t) = (5.6 \text{ m}) \cos[(1.3 \text{ radians/s})t]$**

1. Use this equation:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.3 \text{ N/m}}{1.3 \text{ kg}}} = 1.3 \text{ radians/s}$$

2. Now use this equation:

$$y = A \cos(\omega t) = (5.6 \text{ m}) \cos[(1.3 \text{ radians/s})t]$$

23 **1,250 J**

1. Use this equation:

$$PE = \frac{1}{2} kx^2$$

2. Plug in the numbers:

$$\begin{aligned} PE &= \frac{1}{2} kx^2 \\ &= \frac{1}{2} (100 \text{ N/m})(5.0 \text{ m})^2 \\ &= 1,250 \text{ J} \end{aligned}$$

24 **5,300J**

1. Use this equation:

$$PE = \frac{1}{2} kx^2$$

2. Plug in the numbers:

$$\begin{aligned} PE &= \frac{1}{2} kx^2 \\ &= \frac{1}{2} (250 \text{ N/m})(6.5 \text{ m})^2 \\ &= 5,300 \text{ J} \end{aligned}$$

25 **2.5 s**

1. Use this equation:

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8 \text{ m/s}^2}{1.5 \text{ m}}} = 2.56 \text{ radians/s}$$

2. Now use this equation:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2.56 \text{ radians/s}} = 2.5 \text{ s}$$

26 **3.5 s**

1. Use this equation:

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8 \text{ m/s}^2}{3.0 \text{ m}}} = 1.8 \text{ radians/s}$$

2. Now use this equation:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.8 \text{ radians/s}} = 3.5 \text{ s}$$

27 **$x = (0.5 \text{ m}) \cos [(3.13 \text{ radians/s})t]$**

1. Use this equation:

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8 \text{ m/s}^2}{1.0 \text{ m}}} = 3.13 \text{ radians/s}$$

2. Now use this equation:

$$x = A \cos (\omega t) = (0.5 \text{ m}) \cos [(3.13 \text{ radians/s})t]$$

Note that the motion of a pendulum is not exactly harmonic. Simple harmonic motion is an approximation that describes the motion of a pendulum when the amplitude A is much smaller than the pendulum length L .

28 **$x = (0.75 \text{ m}) \cos [(2.6 \text{ radians/s})t]$**

1. Use this equation:

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8 \text{ m/s}^2}{1.5 \text{ m}}} = 2.6 \text{ radians/s}$$

2. Use this equation and plug in the numbers:

$$x = A \cos (\omega t) = (0.75 \text{ m}) \cos [(2.6 \text{ radians/s})t]$$

4

**Obeying the
Laws of
Thermodynamics**

IN THIS PART . . .

Discover how heat makes items expand.

See how heat is produced, and understand the relationship among the pressure, volume, and temperature of a gas.

Check out the laws of thermodynamics that govern heat processes.

IN THIS CHAPTER

- » Converting between temperature scales
- » Working with linear expansion
- » Calculating volume expansion
- » Using heat capacities
- » Understanding latent heat

Chapter 12

You're Getting Warm: Thermodynamics

Thermodynamics is the study of heat. It's what comes into play when you drop an ice cube into a cup of hot tea and wait to see what happens — if the ice cube or the tea wins out.

In physics, you often run across questions that involve thermodynamics in all sorts of situations. This chapter refreshes your understanding of the topic and lets you put it to use with practice problems that address thermodynamics from all angles.

Converting between Temperature Scales



REMEMBER

You start working with questions of heat by establishing a scale for measuring temperature. The temperature scales that you work with in physics are Fahrenheit, Celsius (sometimes called centigrade), and Kelvin.

Fahrenheit temperatures range from 32° for freezing water to 212° for boiling water. Celsius goes from 0° for freezing water to 100° for boiling water. Following are the equations you use to convert from Fahrenheit (F) temperatures to Celsius (C) and back again:

$$C = \frac{5}{9}(F - 32^\circ)$$

$$F = \frac{9}{5}C + 32^\circ$$

The Kelvin (K) scale is a little different: Its 0 corresponds to *absolute zero*, the temperature at which all molecular motion stops. Absolute zero is at a temperature of -273.15° Celsius, which means that you can convert between Celsius and Kelvin this way:

$$K = C + 273.15$$

$$C = K - 273.15$$

To convert from Kelvin to Fahrenheit degrees, use this formula:

$$F = \frac{9}{5}(K - 273.15) + 32 = \frac{9}{5}K - 459.67$$



Technically, you don't say "degrees Kelvin" but rather "Kelvins," as in 53 Kelvins. However, people persist in using "degrees Kelvin."

REMEMBER



Q. What is 54° Fahrenheit in Celsius?

A. The correct answer is 12°C .

EXAMPLE

1. Use this equation:

$$C = \frac{5}{9}(F - 32^\circ)$$

2. Plug in the numbers:

$$C = \frac{5}{9}(F - 32^\circ) = (0.56)(54^\circ - 32^\circ) = 12^\circ\text{C}$$

1 What is 23° Fahrenheit in Celsius?

2 What is 89° Fahrenheit in Celsius?

3 What is 18° Celsius in Fahrenheit?

4 What is 18° Celsius in Kelvin?

5 What is 18 Kelvins in Celsius?

6 What is 57 Kelvins in Fahrenheit?

Getting Bigger: Linear Expansion

Ever try to open a screw-top jar by running hot water over it? That hot water makes the lid of the jar expand, making it easier to turn. This simple solution is physics on the job — it's all about *thermal expansion*.

You can see an example of thermal expansion in Figure 12-1, where a bar is undergoing expansion in one direction, called *linear expansion*.

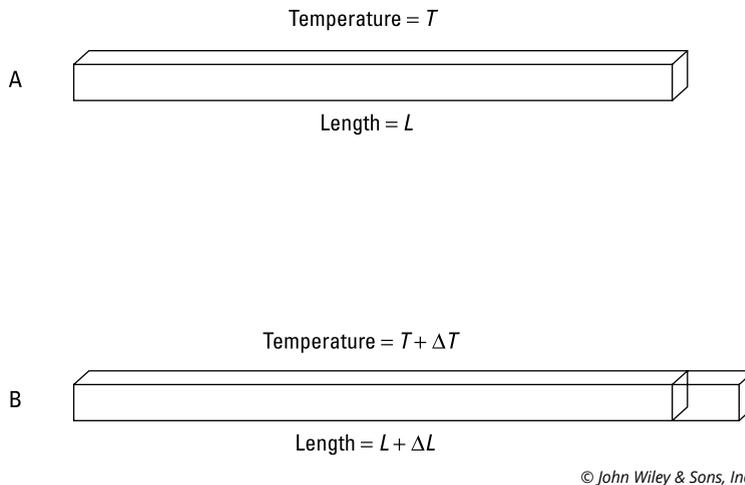


FIGURE 12-1:
Linear
expansion.

When you talk about the expansion of a solid in any one dimension under the influence of heat, you're talking about *linear expansion*. When you raise the temperature a small amount, this equation applies:

$$T_f = T_o + \Delta T$$

Linear expansion results in an expansion in any linear direction of the following:

$$L_f = L_o + \Delta L$$

If the temperature goes down a small amount, this equation applies:

$$T_f = T_o - \Delta T$$

You get a contraction instead of an expansion:

$$L_f = L_o - \Delta L$$

Like the coefficient of friction, a coefficient is in play here — *the coefficient of linear expansion*, which is given the symbol α . So you can write this:

$$\frac{\Delta L}{L_o} = \alpha \Delta T$$

This equation is usually written in this form:

$$\Delta L = \alpha L_o \Delta T$$

Which means:

$$L_f = L_o + \alpha L_o \Delta T$$

Here, α is usually measured in $1 / ^\circ\text{C}$, or $^\circ\text{C}^{-1}$.



EXAMPLE

Q. You're heating a 1.0 m steel bar, coefficient of linear expansion $1.2 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, raising its temperature by 5°C . What is the final length of the bar?

A. The correct answer is 1.00006 m.

1. Use this equation:

$$\Delta L = \alpha L_o \Delta T$$

2. Plug in the numbers:

$$\Delta L = \alpha L_o \Delta T = (1.2 \times 10^{-5} \text{ }^\circ\text{C}^{-1})(1.0 \text{ m})(5 \text{ }^\circ\text{C}) = 6.0 \times 10^{-5} \text{ m} = 0.00006 \text{ m}$$

3. The final length is

$$L_f = L_o + \Delta L = 1.00006 \text{ m}$$

7 You're heating a 1.0 m aluminum bar, coefficient of linear expansion $2.3 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, raising its temperature by 100°C . What is the final length of the bar?

8 You're heating a 2.0 m gold bar, coefficient of linear expansion $1.4 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, raising its temperature by 200°C . What is the final length of the bar?

9 You're heating a 1.5 m copper bar, coefficient of linear expansion $1.7 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, raising its temperature by 300°C . What is the final length of the bar?

10 You're heating a 2.5 m lead bar, coefficient of linear expansion $2.9 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, raising its temperature by 40°C . What is the final length of the bar?

Plumping It Up: Volume Expansion

In addition to linear expansion, physics problems can ask you to find volume expansion. Linear expansion takes place in only one dimension, but volume expansion happens in all three dimensions.

In other words, you have this:

$$\frac{\Delta V}{V_o} \text{ (fraction the solid expands) is proportional to } \Delta T \text{ (change in temperature)}$$

The constant involved in volume expansion is called the *coefficient of volume expansion*. This constant is given by the symbol β , and like α , it's measured in $^{\circ}\text{C}^{-1}$. Using β , here's how you can express the relationship shown in the preceding equation:

$$\frac{\Delta V}{V_o} = \beta \Delta T$$

When you multiply both sides by V_o , you're left with the following:

$$\Delta V = \beta V_o \Delta T$$

Which means:

$$V_f = V_o + \beta V_o \Delta T$$



Q. You're heating a 1.0 m^3 steel block, coefficient of volume expansion $3.6 \times 10^{-5} \text{ }^{\circ}\text{C}^{-1}$, raising its temperature by 45°C . What is the final volume of the bar?

EXAMPLE

A. The correct answer is 1.0016 m^3 .

1. Use this equation:

$$\Delta V = \beta V_o \Delta T$$

2. Plug in the numbers:

$$\Delta V = \beta V_o \Delta T = (3.6 \times 10^{-5} \text{ }^{\circ}\text{C}^{-1})(1.0 \text{ m}^3)(45 \text{ }^{\circ}\text{C}) = 1.6 \times 10^{-3} \text{ m}^3 = 0.0016 \text{ m}^3$$

3. The final volume is

$$V_f = V_o + \beta V_o \Delta T = 1.0016 \text{ m}^3$$

11 You're heating a 2.0 m^3 aluminum block, coefficient of volume expansion $6.9 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, raising its temperature by 30°C . What is the final volume of the block?

12 You're heating a 2.0 m^3 copper block, coefficient of volume expansion $5.1 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, raising its temperature by 20°C . What is the final volume of the block?

13 You're heating a 1.0 m^3 glass block, coefficient of volume expansion $1.0 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, raising its temperature by 27°C . What is the final volume of the block?

14 You're heating a 3.0 m^3 gold block, coefficient of volume expansion $4.2 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, raising its temperature by 18°C . What is the final volume of the block?

Getting Specific with Heat Capacity

It's a fact of physics that it takes 4,186 J to raise the temperature of 1.0 kg of water by 1°C . But it takes only 840 J to raise the temperature of 1.0 kg of glass by 1°C .

You can relate the amount of heat, Q , it takes to raise the temperature of an object to the change in temperature and the amount of mass involved. Use this equation:

$$Q = mc\Delta T$$

In this equation, Q is the amount of heat energy involved (measured in Joules if you're using the MKS system), m is the amount of mass, ΔT is the change in temperature, and c is a constant called the specific heat capacity, which is measured in $J/(kg\cdot^{\circ}C)$ in the MKS system.



REMEMBER

So it takes 4,186 J of heat to warm up 1.0 kg of water 1.0°C. One calorie is defined as the amount of heat needed to heat 1.0 g of water 1.0°C, so 1 calorie equals 4.186 J. Nutritionists use the food energy term Calorie (capital C) to stand for 1,000 calories, 1.0 kcal, so 1.0 Calorie equals 4,186 J. And when you're speaking in terms of heat, you have another unit of measurement to deal with: the British thermal unit (BTU). 1.0 BTU is the amount of heat needed to raise one pound of water 1.0°F and is a unit often used to measure energy in the HVAC industry. To convert BTUs to Joules, use the relation that 1 BTU equals 1,055 J.

If you heat an object, raising its temperature from T_o to T_f , the amount of heat you need is expressed as:

$$\Delta Q = mc(T_f - T_o)$$



EXAMPLE

Q. You're heating a 1.0 kg copper block, specific heat capacity of 387 $J/(kg\cdot^{\circ}C)$, raising its temperature by 45°C. What amount of heat do you have to apply?

A. The correct answer is 17,400 J.

1. Use this equation:

$$Q = mc\Delta T$$

2. Plug in the numbers:

$$Q = mc\Delta T = (387 J/(kg\cdot^{\circ}C))(1.0 kg)(45^{\circ}C) = 17,400 J$$

15 You're heating a 15.0 kg copper block, specific heat capacity of 387 $J/(kg\cdot^{\circ}C)$, raising its temperature by 100°C. What heat do you have to apply?

16 You're heating a 10.0 kg steel block, specific heat capacity of 562 $J/(kg\cdot^{\circ}C)$, raising its temperature by 170°C. What heat do you have to apply?

17 You're heating a 3.0 kg glass block, specific heat capacity of $840 \text{ J}/(\text{kg}\cdot^\circ\text{C})$, raising its temperature by 60°C . What heat do you have to apply?

18 You're heating a 5.0 kg lead block, specific heat capacity of $128 \text{ J}/(\text{kg}\cdot^\circ\text{C})$, raising its temperature by 19°C . What heat do you have to apply?

19 You're cooling a 10.0 kg lead block, specific heat capacity of $128 \text{ J}/(\text{kg}\cdot^\circ\text{C})$, lowering its temperature by 60°C . What heat do you have to extract?

20 You're cooling a 80.0 kg glass block, specific heat capacity of $840 \text{ J}/(\text{kg}\cdot^\circ\text{C})$, lowering its temperature by 16°C . What heat do you have to extract?

21 You put 7,600 J into a 14 kg block of silver, specific heat capacity of $235 \text{ J}/(\text{kg}\cdot^\circ\text{C})$. How much have you raised its temperature?

22 You add 10,000 J into an 8.0 kg block of copper, specific heat capacity of $387 \text{ J}/(\text{kg}\cdot^\circ\text{C})$. How much have you raised its temperature?

Changes of Phase: Latent Heat

Heating blocks of lead is fine, but if you heat that lead enough, sooner or later it's going to melt. When it melts, its temperature stays the same until it liquefies, and then the temperature of the lead increases again as you add heat. So why does its temperature stay constant as it melts? Because the heat you applied went into melting the lead. There's a latent heat of melting that means that so many Joules must be applied per kilogram to make lead change phase from solid to liquid.



The units of latent heat are J/kg.

REMEMBER

There are three phase changes that matter can go through — solid, liquid, and gas — and each transition has a latent heat:

- » **Solid to liquid:** The latent heat of melting (or heat of fusion), L_m or L_f , is the heat per kilogram needed to make the change between the solid and liquid phases (such as when ice turns to water).
- » **Liquid to gas:** The latent heat of vaporization, L_v , is the heat per kilogram needed to make the change between the liquid and gas stages (such as when water boils).
- » **Solid to gas:** The latent heat of sublimation, L_s , is the heat per kilogram needed to make the change between the solid and gas phases (such as the direct sublimation of dry ice [CO_2] to the vapor state).

The latent heat of fusion of water is about 3.35×10^5 J/kg. That means it takes 3.35×10^5 J of energy to melt 1 kg of ice.



Q. You have a glass of 50.0 g of water at room temperature, 25°C, but you'd prefer ice water at 0°C. How much ice at 0.0°C do you need to add?

EXAMPLE

A. The correct answer is 15.6 g.

1. The heat absorbed by the melting ice must equal the heat lost by the water you want to cool. Here's the heat lost by the water you're cooling:

$$\Delta Q_{\text{water}} = mc\Delta T = mc(T_f - T_o)$$

2. Plug in the numbers:

$$\begin{aligned}\Delta Q_{\text{water}} &= mc\Delta T \\ &= mc(T_f - T_o) \\ &= (0.050 \text{ kg})(4,186 \text{ J/kg}\cdot^\circ\text{C})(0^\circ\text{C} - 25^\circ\text{C}) \\ &= -5.23 \times 10^3 \text{ J}\end{aligned}$$

3. So the water needs to lose 5.23×10^3 J. How much ice would that melt? That looks like this, where L_m is the latent heat of melting:

$$\Delta Q_{\text{ice}} = m_{\text{ice}}L_m$$

4. You know that for water, L_m is 3.35×10^5 J/kg, so you get this:

$$\Delta Q_{ice} = m_{ice} L_m = m_{ice} (3.35 \times 10^5 \text{ J/kg})$$

5. You know that equation has to be equal to the heat lost by the water, so you can set it to:

$$\Delta Q_{ice} = \Delta Q_{water}$$

In other words:

$$m_{ice} = \frac{\Delta Q_{water}}{L_m} = \frac{5.23 \times 10^3 \text{ J}}{3.35 \times 10^5 \text{ J/kg}}$$

6. You can now evaluate how much ice needs to be melted:

$$m_{ice} = \frac{5.23 \times 10^3 \text{ J}}{3.35 \times 10^5 \text{ J/kg}} = 1.56 \times 10^{-2} \text{ kg}$$

So you need 1.56×10^{-2} kg, or 15.6 g of ice.

23 You have 100.0 g of coffee in your mug at 80°C . How much ice at 0.0°C would it take to cool 100.0 g of coffee at 80°C to 65°C ?

24 You have 200.0 g of cocoa at 90°C . How much ice at 0.0°C do you have to add to the cocoa (assuming that it has the specific heat capacity of water) to cool it down to 60°C ?

Answers to Problems about Thermodynamics

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

① **-5°C**

1. Use this equation:

$$C = \frac{5}{9}(F - 32^\circ)$$

2. Plug in the numbers:

$$C = \frac{5}{9}(F - 32^\circ) = (0.56)(23^\circ - 32^\circ) = -5^\circ\text{C}$$

② **32°C**

1. Use this equation:

$$C = \frac{5}{9}(F - 32^\circ)$$

2. Plug in the numbers:

$$C = \frac{5}{9}(F - 32^\circ) = (0.56)(89^\circ - 32^\circ) = 32^\circ\text{C}$$

③ **64°F**

1. Use this equation:

$$F = \frac{9}{5}C + 32^\circ$$

2. Plug in the numbers:

$$F = \frac{9}{5}C + 32^\circ = (1.8)(18^\circ) + 32^\circ = 64^\circ\text{F}$$

④ **291 K**

1. Use this equation:

$$K = C + 273.15$$

2. Plug in the numbers:

$$K = C + 273.15 = 18 + 273.15 = 291 \text{ K}$$

⑤ **-255°C**

1. Use this equation:

$$C = K - 273.15$$

2. Plug in the numbers:

$$C = K - 273.15 = 18 - 273.15 = -255^\circ\text{C}$$

6 **-357°F**

1. Convert Kelvins to Celsius:

$$C = K - 273 = 57 - 273 = -216^\circ\text{C}$$

2. Convert Celsius to Fahrenheit:

$$F = \frac{9}{5}C + 32^\circ = (1.8)(-216^\circ) + 32^\circ = -357^\circ\text{F}$$

7 **1.0023 m**

1. Use this equation:

$$\Delta L = \alpha L_o \Delta T$$

2. Plug in the numbers:

$$\Delta L = \alpha L_o \Delta T = (2.3 \times 10^{-5} \text{ }^\circ\text{C}^{-1})(1.0 \text{ m})(100 \text{ }^\circ\text{C}) = 2.3 \times 10^{-3} \text{ m}$$

3. The final length is

$$L_f = L_o + \Delta L = 1.0023 \text{ m}$$

8 **2.0056 m**

1. Use this equation:

$$\Delta L = \alpha L_o \Delta T$$

2. Plug in the numbers:

$$\Delta L = \alpha L_o \Delta T = (1.4 \times 10^{-5} \text{ }^\circ\text{C}^{-1})(2.0 \text{ m})(200 \text{ }^\circ\text{C}) = 5.6 \times 10^{-3} \text{ m}$$

3. The final length is

$$L_f = L_o + \Delta L = 2.0056 \text{ m}$$

9 **1.5077 m**

1. Use this equation:

$$\Delta L = \alpha L_o \Delta T$$

2. Plug in the numbers:

$$\Delta L = \alpha L_o \Delta T = (1.7 \times 10^{-5} \text{ }^\circ\text{C}^{-1})(1.5 \text{ m})(300 \text{ }^\circ\text{C}) = 7.7 \times 10^{-3} \text{ m}$$

3. The final length is

$$L_f = L_o + \Delta L = 1.5077 \text{ m}$$

10 **2.5029 m**

1. Use this equation:

$$\Delta L = \alpha L_o \Delta T$$

2. Plug in the numbers:

$$\Delta L = \alpha L_o \Delta T = (2.9 \times 10^{-5} \text{ }^\circ\text{C}^{-1})(2.5 \text{ m})(40 \text{ }^\circ\text{C}) = 0.0029 \text{ m}$$

3. The final length is

$$L_f = L_o + \Delta L = 2.5029 \text{ m}$$

11 **2.0041 m³**

1. Use this equation:

$$\Delta V = \beta V_o \Delta T$$

2. Plug in the numbers:

$$\Delta V = \beta V_o \Delta T = (6.9 \times 10^{-5} \text{ }^\circ\text{C}^{-1})(2.0 \text{ m}^3)(30 \text{ }^\circ\text{C}) = 0.0041 \text{ m}^3$$

3. The final volume is

$$V_f = V_o + \Delta V = 2.0041 \text{ m}^3$$

12 **2.0020 m³**

1. Use this equation:

$$\Delta V = \beta V_o \Delta T$$

2. Plug in the numbers:

$$\Delta V = \beta V_o \Delta T = (5.1 \times 10^{-5} \text{ }^\circ\text{C}^{-1})(2.0 \text{ m}^3)(20 \text{ }^\circ\text{C}) = 0.0020 \text{ m}^3$$

3. The final volume is

$$V_f = V_o + \Delta V = 2.0020 \text{ m}^3$$

13 **1.00027 m³**

1. Use this equation:

$$\Delta V = \beta V_o \Delta T$$

2. Plug in the numbers:

$$\Delta V = \beta V_o \Delta T = (1.0 \times 10^{-5} \text{ }^\circ\text{C}^{-1})(1.0 \text{ m}^3)(27 \text{ }^\circ\text{C}) = 0.00027 \text{ m}^3$$

3. The final volume is

$$V_f = V_o + \Delta V = 1.00027 \text{ m}^3$$

14 **3.0023 m³**

1. Use this equation:

$$\Delta V = \beta V_o \Delta T$$

2. Plug in the numbers:

$$\Delta V = \beta V_o \Delta T = (4.2 \times 10^{-5} \text{ }^\circ\text{C}^{-1})(3.0 \text{ m}^3)(18 \text{ }^\circ\text{C}) = 0.0023 \text{ m}^3$$

3. The final volume is

$$V_f = V_o + \Delta V = 3.0023 \text{ m}^3$$

15) **$5.8 \times 10^5 \text{ J}$**

1. Use this equation:

$$Q = mc\Delta T$$

2. Plug in the numbers:

$$Q = mc\Delta T = (387 \text{ J}/(\text{kg}\cdot^\circ\text{C}))(15.0 \text{ kg})(100^\circ\text{C}) = 5.8 \times 10^5 \text{ J}$$

16) **$9.6 \times 10^5 \text{ J}$**

1. Use this equation:

$$Q = mc\Delta T$$

2. Plug in the numbers:

$$Q = mc\Delta T = (562 \text{ J}/(\text{kg}\cdot^\circ\text{C}))(10.0 \text{ kg})(170^\circ\text{C}) = 9.6 \times 10^5 \text{ J}$$

17) **$1.5 \times 10^5 \text{ J}$**

1. Use this equation:

$$Q = mc\Delta T$$

2. Plug in the numbers:

$$Q = mc\Delta T = (840 \text{ J}/(\text{kg}\cdot^\circ\text{C}))(3.0 \text{ kg})(60^\circ\text{C}) = 1.5 \times 10^5 \text{ J}$$

18) **$1.2 \times 10^4 \text{ J}$**

1. Use this equation:

$$Q = mc\Delta T$$

2. Plug in the numbers:

$$Q = mc\Delta T = (128 \text{ J}/(\text{kg}\cdot^\circ\text{C}))(5.0 \text{ kg})(19^\circ\text{C}) = 1.2 \times 10^4 \text{ J}$$

19) **76,800 J**

1. Use this equation:

$$Q = mc\Delta T$$

2. Plug in the numbers:

$$Q = mc\Delta T = (128 \text{ J}/(\text{kg}\cdot^\circ\text{C}))(10.0 \text{ kg})(-60^\circ\text{C}) = -76,800 \text{ J}$$

So the heat extracted is 76,800 J.

20 $1.1 \times 10^6 \text{ J}$

1. Use this equation:

$$Q = mc\Delta T$$

2. Plug in the numbers:

$$Q = mc\Delta T = (840 \text{ J/(kg}\cdot\text{°C)})(80.0 \text{ kg})(-16\text{°C}) = -1.1 \times 10^6 \text{ J}$$

So the heat extracted is $1.1 \times 10^6 \text{ J}$.

21 2.3°C

1. Use this equation:

$$Q = mc\Delta T$$

2. Solve for ΔT :

$$\frac{Q}{mc} = \Delta T$$

3. Plug in the numbers:

$$\frac{Q}{mc} = \Delta T = \frac{7,600 \text{ J}}{(235 \text{ J/(kg}\cdot\text{°C)})(14 \text{ kg})} = 2.3\text{°C}$$

22 3.2°C

1. Use this equation:

$$Q = mc\Delta T$$

2. Solve for ΔT :

$$\frac{Q}{mc} = \Delta T$$

3. Plug in the numbers:

$$\frac{Q}{mc} = \Delta T = \frac{10,000 \text{ J}}{(387 \text{ J/(kg}\cdot\text{°C)})(8.0 \text{ kg})} = 3.2\text{°C}$$

23 10.3 g

1. Calculate how much heat has to be lost by the coffee. Assuming it has the same specific heat capacity as water, that's:

$$\Delta Q_{\text{coffee}} = mc\Delta T = mc(T_f - T_o)$$

2. Plug in the numbers:

$$\begin{aligned}\Delta Q_{\text{coffee}} &= mc\Delta T \\ &= mc(T_f - T_o) \\ &= (0.10 \text{ kg})(4,186 \text{ J})(65\text{°C} - 80\text{°C}) \\ &= -6.28 \times 10^3 \text{ J}\end{aligned}$$

3. How much ice do you need to remove 6.28×10^3 J? In this case, the heat supplied to the ice not only must melt the ice:

$$\Delta Q_{ice} = m_{ice}L_m$$

but this heat also needs to raise the temperature of the water that comes from melting the ice from 0°C to 65°C , so you have to add this:

$$\Delta Q_{ice} = m_{ice}L_m + m_{ice}c\Delta T = m_{ice}L_m + m_{ice}c(T_f - T_o)$$

4. This has to be equal to the heat lost by the coffee, so you get

$$6.28 \times 10^3 \text{ J} = m_{ice}[L_m + c(T_f - T_o)]$$

5. Solve for m_{ice} :

$$m_{ice} = \frac{6.28 \times 10^3 \text{ J}}{[L_m + c(T_f - T_o)]}$$

6. Plug in the numbers:

$$\begin{aligned} m_{ice} &= \frac{6.28 \times 10^3 \text{ J}}{[L_m + c(T_f - T_o)]} \\ &= \frac{6.28 \times 10^3 \text{ J}}{[(3.35 \times 10^5 \text{ J/kg}) + (4,186 \text{ J})(65^\circ\text{C} - 0^\circ\text{C})]} \\ &= 0.0103 \text{ kg} \end{aligned}$$

So you need 1.03×10^{-2} kg, or 10.3 g of ice.

24 43 g

1. Find how much heat has to be lost by the cocoa. Assuming it has the same specific heat capacity as water, that's:

$$\Delta Q_{cocoa} = mc\Delta T = mc(T_f - T_o)$$

2. Plug in the numbers:

$$\begin{aligned} \Delta Q_{cocoa} &= mc\Delta T \\ &= mc(T_f - T_o) \\ &= (0.20 \text{ kg})(4,186 \text{ J})(60^\circ\text{C} - 90^\circ\text{C}) \\ &= -2.5 \times 10^4 \text{ J} \end{aligned}$$

3. How much ice do you need to remove 2.5×10^4 J from the cocoa? In this case, the heat supplied to the ice not only must melt the ice:

$$\Delta Q_{ice} = m_{ice}L_m$$

4. But this heat also needs to raise the temperature of the water that comes from melting the ice from 0°C to 60°C , so you have to add this:

$$\Delta Q_{ice} = m_{ice}L_m + m_{ice}c\Delta T = m_{ice}L_m + m_{ice}c(T_f - T_o)$$

5. This has to be equal to the heat lost by the cocoa, so you get

$$2.5 \times 10^4 \text{ J} = m_{ice} [L_m + c(T_f - T_o)]$$

6. Solve for m_{ice} :

$$m_{ice} = \frac{2.5 \times 10^4 \text{ J}}{[L_m + c(T_f - T_o)]}$$

7. Plug in the numbers:

$$\begin{aligned} m_{ice} &= \frac{2.5 \times 10^4 \text{ J}}{[L_m + c(T_f - T_o)]} \\ &= \frac{2.5 \times 10^3 \text{ J}}{[(3.35 \times 10^5 \text{ J/kg}) + (4,186 \text{ J})(60^\circ\text{C} - 0^\circ\text{C})]} \\ &= 0.043 \text{ kg} \end{aligned}$$

So you need 4.3×10^{-2} kg, or 43 g of ice.

IN THIS CHAPTER

- » Handling heat convection and heat conduction
- » Dealing with heat radiation
- » Using Avogadro's number
- » Working with molecular motion

Chapter **13**

Under Pressure: From Solid to Liquid to Gas

Heat is the transfer of thermal energy between objects, usually through conduction, convection, or radiation. *Thermal energy* is the energy associated with the temperature of an object.

This chapter is concerned with all kinds of things heat related — rate of heat, for example, which is about how fast thermal energy travels along iron bars, aluminum pot handles, and more. Want to know how long you can hold that pot over an open flame? This chapter is for you.

You also find here the famous ideal gas law, which connects the pressure, volume, and temperature of a gas. Want to know the pressure of a volume of gas at a particular temperature? Just turn to the ideal gas law, and it'll give you the answer.

Plenty of thermodynamics comes up in this chapter, from how fast thermal energy travels to what pressure gas has at a certain temperature.

In this chapter, you discover the three primary means by which heat is produced: convection, conduction, and radiation.

How Heat Flows: Convection

Take a look at Figure 13-1, where you see a pot of water being heated. The water in that pot moves in the pattern you see in the figure; as it circulates, the hot water moves from the bottom to the top.

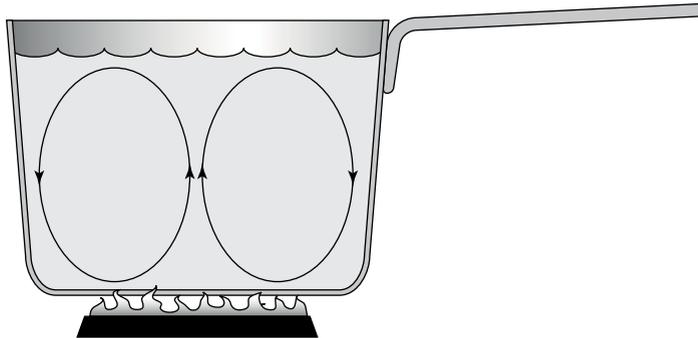


FIGURE 13-1: Convection causes all of the water in the pot to warm.

© John Wiley & Sons, Inc.

That's the first method of moving thermal energy: convection. *Convection* allows thermal energy to move by the motion of heated matter.



EXAMPLE

Q. How does the top of the pot get heated in Figure 13-1?

A. The correct answer is convection.

The top of the pot is heated through convection. The water heats up near the flame and then rises in the pot.

1 How is the top of the inside of a stove heated from the bottom?

2 How is a point 2,000 feet above Earth's surface heated from the surface?

How Heat Is Produced: Conduction

Take a look at Figure 13-2, where you see a metal pot being heated — and the handle is getting warm. The mechanism that produces heat between points on the pot's handle is called *conduction*.

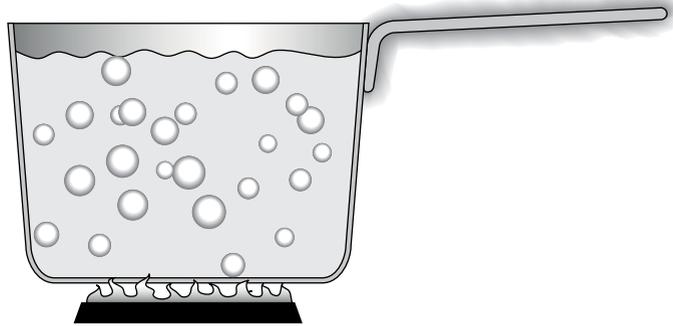


FIGURE 13-2:
Conduction heats the pot that holds the boiling water.

© John Wiley & Sons, Inc.

How much thermal energy travels between two points via conduction? The amount of heat is proportional to the difference in temperature, so for heat conduction between the ends of a bar of metal, you'd have this:

$$Q \propto \Delta T$$

As you may expect, a bar twice as wide conducts twice the amount of heat. In general, the amount of heat conducted is proportional to the cross-sectional area:

$$Q \propto A$$

Also, the longer the bar, the less heat makes it all the way through. In fact, the conducted heat turns out to be inversely proportional to the length of the bar:

$$Q \propto \frac{1}{L}$$

Finally, the amount of heat depends on the amount of time that passes — twice the time, twice the heat. That makes sense too. Here's how you express that mathematically:

$$Q \propto t$$

So here's what you get when you put it all together (k is some constant yet to be determined):

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

What is the constant k ? This constant is called the thermal conductivity of a material, and its units are $J/(s \cdot m \cdot ^\circ C)$.



EXAMPLE

Q. The thermal conductivity of copper is $390 \text{ J}/(\text{s}\cdot\text{m}\cdot^\circ\text{C})$. How much heat is conducted per second between two points on a copper rod, cross section 0.10 m^2 , 1.0 m apart, with a temperature difference of 69°C ?

A. The correct answer is $2,700 \text{ J}/\text{s}$.

1. Use this equation:

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

2. Solve for Q/t :

$$\frac{Q}{t} = \frac{k \cdot A \cdot \Delta T}{L}$$

3. Plug in the numbers:

$$\begin{aligned} \frac{Q}{t} &= \frac{k \cdot A \cdot \Delta T}{L} \\ &= \frac{(390 \text{ J}/(\text{s}\cdot\text{m}\cdot^\circ\text{C}))(0.10 \text{ m}^2)(69^\circ\text{C})}{1.0 \text{ m}} \\ &= 2,690 \text{ J}/\text{s} \end{aligned}$$

That answer rounds to $2,700 \text{ J}/\text{s}$ with significant figures.

3 The thermal conductivity of steel is $79 \text{ J}/(\text{s}\cdot\text{m}\cdot^\circ\text{C})$. How much heat is conducted per second between two points on a steel rod, cross section 0.3 m^2 , 2.0 m apart, with a temperature difference of 34°C ?

4 The thermal conductivity of silver is $420 \text{ J}/(\text{s}\cdot\text{m}\cdot^\circ\text{C})$. How much heat is conducted per second between two points on a silver rod, cross section 0.40 m^2 , 1.5 m apart, with a temperature difference of 91°C ?

5 If you have a steel rod, thermal conductivity $79 \text{ J}/(\text{s}\cdot\text{m}\cdot^\circ\text{C})$, with a cross section 0.30 m^2 and a length of 50 cm , how long do you have to wait for $1,000.0 \text{ J}$ to be transferred from one end to the other if the temperature difference is 90°C ?

6 If you have a brass rod, thermal conductivity $110 \text{ J}/(\text{s}\cdot\text{m}\cdot^\circ\text{C})$, with a cross section 0.50 m^2 and a length of 73 cm , how long do you have to wait for $3,000.0 \text{ J}$ to be transferred from one end to the other if the temperature difference is 180°C ?

7 You have a mystery substance, length 2.0 m , cross section 0.40 m^2 , and apply a temperature difference of 69°C across its length. If the substance conducts $1,000.0 \text{ J}$ of heat in 0.50 s , what is its thermal conductivity?

8 You have an unknown material, length 1.0 m , cross section 0.30 m^2 , and apply a temperature difference of 97°C across its length. If the substance conducts $1,400.0 \text{ J}$ of heat in 0.60 s , what is its thermal conductivity?

How Heat Is Produced: Radiation

The third way that heat is produced, after convection and conduction, is *radiation*. Anything hot radiates thermal energy in the form of electromagnetic radiation, which is the way the sun warms Earth. The sun can't warm Earth due to convection or conduction, because a vacuum exists between here and there. Instead, the sun sends light to Earth in a wide range of frequencies, and that energy warms Earth's surface.

The amount of heat produced this way is proportional to the amount of time that the radiant object beams energy:

$$Q \propto t$$

As you may also expect, the amount of heat radiated is proportional to the total area doing the radiating — twice as much area doing the radiating, twice as much heat radiated. So you can write this equation, where A is the area doing the radiating:

$$Q \propto A \cdot t$$

You'd expect temperature, T , to be in here somewhere — the hotter an object, the more heat radiated. Hold on to your hat: It turns out that the amount of heat radiated is proportional to T in Kelvins to the fourth power, T^4 . So now you have

$$Q \propto A \cdot t \cdot T^4$$

To make this relationship an equation, all you need to add is a constant, which is measured experimentally. This constant, called the Stefan-Boltzmann constant, σ , goes in like this:

$$Q = \sigma \cdot A \cdot t \cdot T^4$$

The value of σ is $5.67 \times 10^{-8} \text{ J}/(\text{s} \cdot \text{m}^2 \cdot \text{K}^4)$.

This equation works only for objects that are so-called perfect emitters, however. Most objects aren't perfect emitters, so you have to add another constant that depends on the substance you're working with. This constant is called its *emissivity*, e . So this equation, the Stefan-Boltzmann law of radiation, becomes

$$Q = e \cdot \sigma \cdot A \cdot t \cdot T^4$$

An object's emissivity depends on what it's made of, so you can expect emissivity to vary from material to material. Emissivity is always between 0 and 1.



EXAMPLE

Q. A person has a surface area of 1.8 m^2 , an emissivity of 0.65, and a temperature of 37°C . How much power does this person radiate?

A. The correct answer is 610 W.

1. Use this equation:

$$Q = e \cdot \sigma \cdot A \cdot t \cdot T^4$$

2. Solve for power, Q/t :

$$\frac{Q}{t} = e \cdot \sigma \cdot A \cdot T^4$$

3. Plug in the numbers, remembering to convert from Celsius to Kelvin:

$$\begin{aligned}\frac{Q}{t} &= e \cdot \sigma \cdot A \cdot T^4 \\ &= (0.65)(5.67 \times 10^{-8} \text{ J/(s}\cdot\text{m}^2\cdot\text{K}^4))(1.8 \text{ m}^2)(37 \text{ K} + 273 \text{ K})^4 \\ &= 610 \text{ W}\end{aligned}$$

9 A coffeepot has a temperature of 90°C and a surface area of 0.20 m^2 . If its emissivity is 0.95, what power is it radiating?

10 A toaster has a temperature of 120°C and a surface area of 0.15 m^2 . If its emissivity is 0.90, what power is it radiating?

11 If a 200 g coffeepot holds 300 g of coffee, its temperature is 80°C , and its surface area is 0.18 m^2 , how long will it take for the coffee to cool to 75°C , given that the pot's emissivity is 0.90 (omitting the fact that the pot is heated by radiation coming from its surroundings)?

12 If a 100 g teacup holds 200 g of tea, its temperature is 82°C , and its surface area is 0.07 m^2 , how long will it take for the tea to cool to 76°C , given that the cup's emissivity is 0.60 and the specific heat capacity of the cup is $840 \text{ J/(kg}\cdot^\circ\text{C)}$ (omitting the fact that the cup is heated by radiation coming from its surroundings)?

A Biggie: Avogadro's Number

A certain amount of mass contains a specific number of atoms, and you can figure out just how many atoms a certain amount of mass has. In particular, a mole is defined as the number of atoms in 12.0 g of carbon isotope 12. (Carbon isotope 12, also called carbon-12 or just carbon 12, is the most common version of carbon, although some carbon atoms have a few more neutrons in them, so the average works out to about 12.011.)

That number of atoms has been measured as 6.022×10^{23} , which is called Avogadro's number, N_A . That number represents many atoms. Now you know how many atoms are in 12.0 g of carbon 12.

Does 12.0 g of sulfur have the same number of atoms? No. Just check a periodic table of the elements. You find that the atomic mass of sulfur is 32.06. But 32.06 what? It turns out to mean 32.06 atomic units, u, where each atomic unit is 1/12 of the mass of a carbon 12 atom. So if a mole of carbon 12 has a mass of 12.0 g, and the mass of an average sulfur atom is bigger than the mass of a carbon 12 atom by this ratio

$$\frac{\text{sulfur mass}}{\text{Carbon 12 mass}} = \frac{32.06 \text{ u}}{12 \text{ u}}$$

a mole of sulfur atoms must have this mass:

$$\frac{32.06 \text{ u}}{12 \text{ u}}(12.0 \text{ g}) = 32.06 \text{ g}$$

Note that sulfur and carbon are composed of simple atoms; if you're dealing with a composite substance such as water, you have to think in terms of molecules instead. So instead of the atomic mass in cases like these, you look for the molecular mass (when atoms combine, you have molecules), which is also measured in atomic mass units. A molecule of water is made up of two hydrogen atoms and one oxygen atom. The molecular mass of hydrogen is 1.0083 u, and the molecular mass of oxygen is 15.9994 u. Therefore, the molecular mass of water is $2(1.0083 \text{ u}) + 15.9994 \text{ u}$, which equals 18.0153 u. So 1 mole of water molecules has a mass of 18.0153 g.



EXAMPLE

Q. How many molecules are in 10.0 g of water?

A. The correct answer is 3.3×10^{23} molecules.

1. You know that 1 mole of water has this mass:

$$1 \text{ mole of water} = 18.0 \text{ g}$$

2. A mole has 6.022×10^{23} molecules, so 10.0 g has this many molecules:

$$6.022 \times 10^{23} \frac{(10.0 \text{ g})}{(18.0 \text{ g})} = 3.3 \times 10^{23}$$

13 You have 10.0 g of calcium, atomic mass 40.08 u. How many atoms do you have?

14 You have 16.0 g of silicon, atomic mass 28.09 u. How many atoms do you have?

15 You have 29.0 g of zinc, atomic mass 65.41 u. How many atoms do you have?

16 You have 3.0 g of copper, atomic mass 63.546 u. How many atoms do you have?

Ideally Speaking: The Ideal Gas Law

You can relate the pressure, volume, and temperature of an ideal gas with the ideal gas equation. An *ideal gas* is one whose molecules act like points; no interaction occurs among molecules except *elastic collisions* (that is, where kinetic energy is conserved). In practice, all gases act like ideal gases to some extent, so the ideal gas law holds fairly well. Here's that law:

$$PV = nRT$$

Here, P is pressure; n is the number of moles of gas you have; R is the *universal gas constant*, which has a value of $8.31 \text{ J}/(\text{mole}\cdot\text{K})$; and T is measured in Kelvins. The volume, V , is measured in cubic meters, m^3 . Sometimes, you'll see volume given in liters, where $1.0 \text{ L} = 10^{-3} \text{ m}^3$. Using this law, you can predict the pressure of a gas, given how much you have of it, its temperature, and the volume you've enclosed it in.

One mole of ideal gas takes up 22.4 L of volume at 0°C and one atmosphere pressure, which is the pressure from the Earth's atmosphere at sea level. A pressure of atmosphere is $1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$, where N/m^2 (Newtons per square meter) is given its own units, Pascals, abbreviated Pa.

You can also write the ideal gas law a little differently by using Avogadro's number, N_A , and the total number of molecules, N :

$$PV = nRT = (N/N_A)RT$$

The constant R/N_A is also called *Boltzmann's constant*, k , and it has a value of $1.38 \times 10^{-23} \text{ J/K}$. Using this constant, the ideal gas law becomes

$$PV = NkT$$

The *gauge pressure* is the difference between the pressure inside a system and the pressure outside (which is usually atmospheric pressure). For example, an ordinary tire gauge measures the difference between the air pressure inside the tire and the air pressure outside the tire.



Q. You have 1.0 moles of air in your tire at 0°C, volume 10.0 L. What is the gauge pressure of the tire?

EXAMPLE

A. The correct answer is $1.3 \times 10^5 \text{ Pa}$.

1. Use this equation:

$$PV = nRT$$

2. Solve for P :

$$P = nRT / V$$

3. Plug in the numbers to get the pressure pushing out:

$$P = nRT / V = (1.0)(8.31 \text{ J/K})(273 \text{ K}) / (10 \times 10^{-3} \text{ m}^3) = 2.3 \times 10^5 \text{ Pa}$$

4. Subtract the pressure from the surrounding air, which pushes in, assuming that the air is at 0°C too:

$$P = 2.3 \times 10^5 \text{ Pa} - 1.013 \times 10^5 \text{ Pa} = 1.3 \times 10^5 \text{ Pa}$$

- 17 You have 2.3 moles of air in your tire at 0°C, volume 12.0 L. What pressure is the tire inflated to?

- 18 You have a bottle of 2.0 moles of gas, volume 1.0 L, temperature 100°C. What is the pressure inside the bottle?

Molecules in Motion

The average kinetic energy of molecules in a gas is

$$KE_{avg} = \frac{3}{2}kT$$

This includes only the energy associated with linear motion of the molecules, and not any energy associated with rotation or vibration. The constant k is Boltzmann's constant, 1.38×10^{-23} J/K. Now you can determine the average kinetic energies of the molecules in a gas. And because you can determine the mass of each molecule if you know what gas you're dealing with, you can figure out their average speeds at various temperatures.



EXAMPLE

Q. What is the speed of nitrogen molecules at 28°C?

A. The correct answer is 518 m/s.

1. Use this equation:

$$KE_{avg} = \frac{3}{2}kT$$

2. Plug in the numbers:

$$\begin{aligned} KE_{avg} &= \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(301 \text{ K}) \\ &= 6.23 \times 10^{-21} \text{ J} \end{aligned}$$

3. You know:

$$KE = \frac{1}{2}mv^2$$

4. Solve for v :

$$v = \sqrt{\frac{2KE}{m}}$$

5. Plug in the numbers. Each nitrogen atom has a mass of about 4.65×10^{-26} kg:

$$\begin{aligned} v &= \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(6.23 \times 10^{-21} \text{ J})}{4.65 \times 10^{-26} \text{ kg}}} = 518 \text{ m/s} \\ &= 1,160 \text{ mph} \end{aligned}$$

Note: Not all nitrogen molecules will have the same speed. Some molecules will have greater kinetic energy, and thus higher speed, and others will have less. 518 m/s is the speed that corresponds to the average kinetic energy.

19 What is the speed of air molecules at 43°C ?

20 What is the speed of air molecules at 900°C ?

Answers to Problems about Pressure

The following are the answers to the practice questions presented in this chapter. You see how to work out each answer, step by step.

1 Convection

The air is heated by the heating elements on the bottom, and the heated air rises, so the transfer of thermal energy takes place via convection.

2 Convection

The air is heated by being in proximity to the heated Earth, and the heated air rises.

3 403 J/s

1. Use this equation:

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

2. Solve for Q/t :

$$\frac{Q}{t} = \frac{k \cdot A \cdot \Delta T}{L}$$

3. Plug in the numbers:

$$\frac{Q}{t} = \frac{k \cdot A \cdot \Delta T}{L} = \frac{(79 \text{ J/(s}\cdot\text{m}\cdot\text{°C)})(0.3 \text{ m}^2)(34\text{°C})}{2.0 \text{ m}} = 403 \text{ J/s}$$

4 10,000 J/s

1. Use this equation:

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

2. Solve for Q/t :

$$\frac{Q}{t} = \frac{k \cdot A \cdot \Delta T}{L}$$

3. Plug in the numbers:

$$\frac{Q}{t} = \frac{k \cdot A \cdot \Delta T}{L} = \frac{(420 \text{ J/(s}\cdot\text{m}\cdot\text{°C)})(0.40 \text{ m}^2)(91\text{°C})}{1.5 \text{ m}} = 10,000 \text{ J/s}$$

5 0.23 s

1. Use this equation:

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

2. Solve for t :

$$t = \frac{L \cdot Q}{k \cdot A \cdot \Delta T}$$

3. Plug in the numbers:

$$t = \frac{L \cdot Q}{k \cdot A \cdot \Delta T} = \frac{(0.5 \text{ m})(1,000.0 \text{ J})}{(79 \text{ J}/(\text{s}\cdot\text{m}\cdot^\circ\text{C}))(0.30 \text{ m}^2)(90^\circ\text{C})} = 0.23 \text{ s}$$

6 **0.22 s**

1. Use this equation:

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

2. Solve for t :

$$t = \frac{L \cdot Q}{k \cdot A \cdot \Delta T}$$

3. Plug in the numbers:

$$t = \frac{L \cdot Q}{k \cdot A \cdot \Delta T} = \frac{(0.73 \text{ m})(3,000.0 \text{ J})}{(110 \text{ J}/(\text{s}\cdot\text{m}\cdot^\circ\text{C}))(0.50 \text{ m}^2)(180^\circ\text{C})} = 0.22 \text{ s}$$

7 **145 J/(s·m·°C)**

1. Use this equation:

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

2. Solve for k :

$$k = \frac{L \cdot Q}{t \cdot A \cdot \Delta T}$$

3. Plug in the numbers:

$$k = \frac{L \cdot Q}{t \cdot A \cdot \Delta T} = \frac{(2.0 \text{ m})(1,000.0 \text{ J})}{(0.5 \text{ s})(0.4 \text{ m}^2)(69^\circ\text{C})} = 145 \text{ J}/(\text{s}\cdot\text{m}\cdot^\circ\text{C})$$

8 **80 J/(s·m·°C)**

1. Use this equation:

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

2. Solve for k :

$$k = \frac{L \cdot Q}{t \cdot A \cdot \Delta T}$$

3. Plug in the numbers:

$$k = \frac{L \cdot Q}{t \cdot A \cdot \Delta T} = \frac{(1.0 \text{ m})(1,400.0 \text{ J})}{(0.60 \text{ s})(0.30 \text{ m}^2)(97^\circ\text{C})} = 80 \text{ J}/(\text{s}\cdot\text{m}\cdot^\circ\text{C})$$

9 190 W

1. Use this equation:

$$Q = e \cdot \sigma \cdot A \cdot t \cdot T^4$$

2. Solve for power, which is Q/t :

$$\frac{Q}{t} = e \cdot \sigma \cdot A \cdot T^4$$

3. Plug in the numbers:

$$\frac{Q}{t} = e \cdot \sigma \cdot A \cdot T^4 = (0.95)(5.67 \times 10^{-8} \text{ J/(s}\cdot\text{m}^2\cdot\text{K}^4))(0.2 \text{ m}^2)(90 \text{ K} + 273 \text{ K})^4 = 190 \text{ J/s} = 190 \text{ W}$$

10 180 W

1. Use this equation:

$$Q = e \cdot \sigma \cdot A \cdot t \cdot T^4$$

2. Solve for power, which is Q/t :

$$\frac{Q}{t} = e \cdot \sigma \cdot A \cdot T^4$$

3. Plug in the numbers:

$$\frac{Q}{t} = e \cdot \sigma \cdot A \cdot T^4 = (0.90)(5.67 \times 10^{-8} \text{ J/(s}\cdot\text{m}^2\cdot\text{K}^4))(0.15 \text{ m}^2)(120 \text{ K} + 273 \text{ K})^4 = 180 \text{ J/s} = 180 \text{ W}$$

11 29 s

1. Use this equation (which works because the temperature difference involved, 80°C to 75°C , is small):

$$Q = e \cdot \sigma \cdot A \cdot t \cdot T^4$$

2. Solve for time:

$$t = \frac{Q}{e\sigma AT^4}$$

3. Solve for the amount of heat you need to lose:

$$\Delta Q = cm\Delta T$$

4. Plug in the numbers, assuming that coffee has the specific heat of water, and ignoring the specific heat of the pot:

$$\Delta Q = cm\Delta T = (4,186 \text{ J/(kg}\cdot^\circ\text{C)})(0.2 \text{ kg})(80^\circ\text{C} - 75^\circ\text{C}) = 4,200 \text{ J}$$

5. Put in the numbers to find the time:

$$t = \frac{Q}{e\sigma AT^4} = \frac{4,200 \text{ J}}{(0.90)(5.67 \times 10^{-8} \text{ J/(s}\cdot\text{m}^2\cdot\text{K}^4))(0.18 \text{ m}^2)(80 \text{ K} + 273 \text{ K})^4} = 29 \text{ s}$$

12 **146 s**

1. Use this equation (which works because the temperature difference involved, 82°C to 76°C , is small):

$$Q = e \cdot \sigma \cdot A \cdot t \cdot T^4$$

2. Solve for time:

$$t = \frac{Q}{e\sigma AT^4}$$

3. Solve for the amount of heat you need to lose:

$$\Delta Q = \text{heat lost by tea} + \text{heat lost by teapot}$$

4. Plug in the numbers, assuming that tea has the specific heat of water, and using the given value for the specific heat capacity of the cup:

$$\begin{aligned} \Delta Q = m_1c_1\Delta T + m_2c_2\Delta T &= (4,186 \text{ J}/(\text{kg}\cdot^\circ\text{C}))(0.200 \text{ kg})(82^\circ\text{C} - 76^\circ\text{C}) + \\ & (840 \text{ J}/(\text{kg}\cdot^\circ\text{C}))(0.100 \text{ kg})(82^\circ\text{C} - 76^\circ\text{C}) = 5,530 \text{ J} \end{aligned}$$

5. Put in the numbers to find the time:

$$t = \frac{Q}{e\sigma AT^4} = \frac{5,530 \text{ J}}{(0.60)(5.67 \times 10^{-8} \text{ J}/(\text{s}\cdot\text{m}^2\cdot\text{K}^4))(0.07 \text{ m}^2)(82 \text{ K} + 273 \text{ K})^4} = 146 \text{ s}$$

13 **1.5×10^{23} atoms**

1. You know that 1 mole of calcium has a mass of 40.08 g, so you have this many moles:

$$n = \frac{10.0 \text{ g}}{40.08 \text{ g}} = 0.25 \text{ moles}$$

2. With this many moles, you have this many molecules:

$$N = nN_A = 0.25(6.022 \times 10^{23}) = 1.5 \times 10^{23} \text{ atoms}$$

14 **3.4×10^{23} atoms**

1. You know that 1 mole of silicon has a mass of 28.09 g, so you have this many moles:

$$n = \frac{16.0 \text{ g}}{28.09 \text{ g}} = 0.57 \text{ moles}$$

2. With this many moles, you have this many molecules:

$$N = nN_A = 0.57(6.022 \times 10^{23}) = 3.4 \times 10^{23} \text{ atoms}$$

15 **2.6×10^{23} atoms**

1. You know that 1 mole of zinc has a mass of 65.41 g, so you have this many moles:

$$n = \frac{29.0 \text{ g}}{65.41 \text{ g}} = 0.44 \text{ moles}$$

2. With this many moles, you have this many molecules:

$$N = nN_A = 0.44(6.022 \times 10^{23}) = 2.6 \times 10^{23} \text{ atoms}$$

16 2.8×10^{22} atoms

1. You know that 1 mole of copper has a mass of 63.546 g, so you have this many moles:

$$n = \frac{3.0 \text{ g}}{63.546 \text{ g}} = 0.047 \text{ moles}$$

2. With this many moles, you have this many molecules:

$$N = nN_A = 0.047(6.022 \times 10^{23}) = 2.8 \times 10^{22} \text{ atoms}$$

17 3.3×10^5 Pa

1. Use this equation:

$$PV = nRT$$

2. Solve for P:

$$P = nRT / V$$

3. Plug in the numbers:

$$P = nRT / V = (2.3)(8.31 \text{ J/K})(273 \text{ K}) / (12.0 \times 10^{-3} \text{ m}^3) = 4.3 \times 10^5 \text{ Pa}$$

4. Subtract the pressure from the surrounding air, which pushes in, assuming that the air is at 0°C too:

$$P = 4.3 \times 10^5 \text{ Pa} - 1.013 \times 10^5 \text{ Pa} = 3.3 \times 10^5 \text{ Pa}$$

18 6.2×10^6 Pa

1. Use this equation:

$$PV = nRT$$

2. Solve for P:

$$P = nRT / V$$

3. Plug in the numbers:

$$P = nRT / V = (2.0)(8.31 \text{ J/K})(373 \text{ K}) / (1.0 \times 10^{-3} \text{ m}^3) = 6.2 \times 10^6 \text{ Pa}$$

19 530 m/s

1. Use this equation:

$$KE_{avg} = \frac{3}{2}kT$$

2. Plug in the numbers:

$$KE_{avg} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K} + 43 \text{ K}) = 6.5 \times 10^{-21} \text{ J}$$

3. Here's the equation for kinetic energy:

$$KE = \frac{1}{2}mv^2$$

4. Solve for v :

$$v = \sqrt{\frac{2KE}{m}}$$

5. Plug in the numbers, assuming the molecular mass of nitrogen, $m = 4.65 \times 10^{-26}$ kg (which you can figure out by using Avogadro's number and the mass of 1 mole of nitrogen):

$$v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(6.5 \times 10^{-21} \text{ J})}{4.65 \times 10^{-26} \text{ kg}}} = 530 \text{ m/s}$$

20 **1,020 m/s**

1. Use this equation:

$$KE_{avg} = \frac{3}{2}kT$$

2. Plug in the numbers:

$$KE_{avg} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23})(273 + 900) = 2.4 \times 10^{-20} \text{ J}$$

3. Here's the equation for kinetic energy:

$$KE = \frac{1}{2}mv^2$$

4. Solve for v :

$$v = \sqrt{\frac{2KE}{m}}$$

5. Plug in the numbers, assuming the molecular mass of nitrogen, $m = 4.65 \times 10^{-26}$ kg (which you can figure out by using Avogadro's number and the mass of 1 mole of nitrogen):

$$v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(2.4 \times 10^{-20} \text{ J})}{4.65 \times 10^{-26} \text{ kg}}} = 1,020 \text{ m/s}$$

- » Understanding the laws of thermodynamics
- » Working with isobaric processes
- » Handling isochoric processes
- » Calculating isothermal processes

Chapter 14

All about Heat and Work

This chapter is all about the laws of thermodynamics. Those laws describe all kinds of heat processes. You also discover how to handle heat processes where pressure is constant, or volume is constant, or temperature is constant.

The First Law of Thermodynamics

The first law of thermodynamics says that energy is conserved. The internal energy in a system, U_o , changes to a final value U_f when heat Q is absorbed or released by the system and the system does work W on its surroundings, or the surroundings do work W on the system, such that:

$$U_f - U_o = \Delta U = Q - W$$

The value Q is positive when the system absorbs heat and negative when the system releases heat. The quantity W is positive when the system does work on its surroundings and negative when the surroundings do work on the system.

This law is useful because it says that total energy — internal energy plus heat — is conserved.



EXAMPLE

Q. Say that a motor does 1,000 J of work on its surroundings while releasing 3,000 J of heat. By how much does its internal energy change?

A. The correct answer is $-4,000$ J.

1. You know the motor does 1,000 J of work on its surroundings, so you know its internal energy decreases by 1,000 J.
2. The motor also releases 3,000 J of heat while doing its work, so the internal energy of the system decreases by an additional 3,000 J. Think of negative values of work and heat as energy flowing out of the system, making the total change of internal energy this:

$$\Delta U = -1,000 - 3,000 = -4,000 \text{ J}$$

1 You have a motor that absorbs 3,000 J of heat while doing 2,000 J of work. What is the change in the motor's internal energy?

2 You have a motor that absorbs 2,500 J of heat while doing 1,700 J of work. What is the change in the motor's internal energy?

Constant Pressure: Isobaric Processes

Take a look at Figure 14-1, in which a lid with a weight on it keeps constant pressure on a gas as that gas expands. Work that's done at constant pressure is called *isobaric work*.

So the question is: What work is the system doing as it expands? $Work = Fs$, and $F = PA$, where P is the pressure and A is the area. That means that:

$$W = F\Delta s = PA\Delta s$$

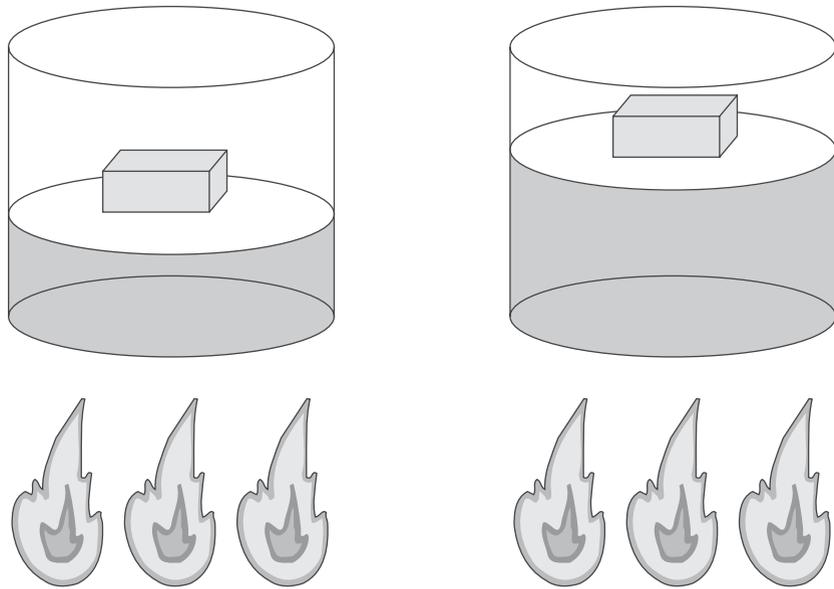


FIGURE 14-1:
An isobaric
system.

© John Wiley & Sons, Inc.

On the other hand, $\Delta s = \Delta V$, the change in volume, so you have

$$W = P\Delta V$$

You can see what this looks like graphically for an isobaric process in Figure 14-2, in which the volume is changing while the pressure stays constant. Because $W = F\Delta V$, the work is the area beneath the graph as shown in the figure.

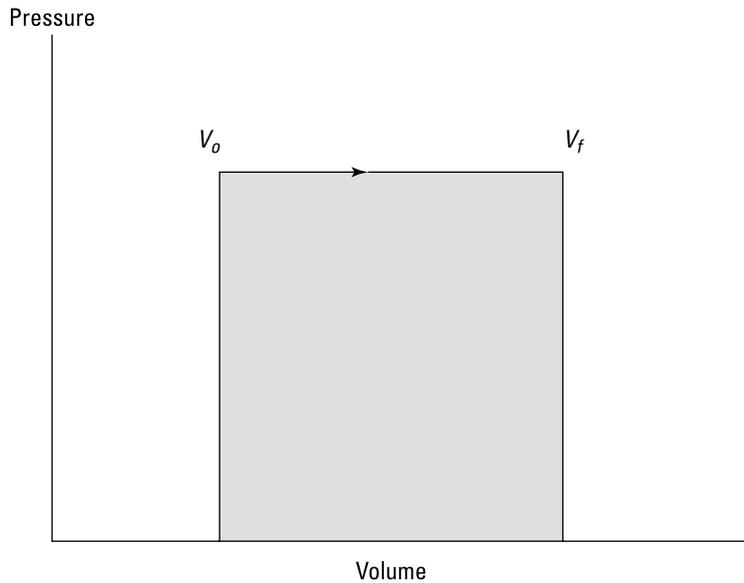


FIGURE 14-2:
An isobaric
graph.

© John Wiley & Sons, Inc.



EXAMPLE

Q. You have 60.0 m^3 of an ideal gas at 200.0 Pa and heat the gas until it expands to a volume of 120 m^3 . How much work did the gas do?

A. The correct answer is $12,000 \text{ J}$.

1. Use this equation:

$$W = P \cdot \Delta V$$

2. Plug in the numbers:

$$W = P \cdot \Delta V = (200.0 \text{ Pa})(120 \text{ m}^3 - 60 \text{ m}^3) = 12,000 \text{ J}$$

The gas did $12,000 \text{ J}$ of work as it expanded under constant pressure.

3 You have 50.0 m^3 of an ideal gas at $1,000.0 \text{ Pa}$ and heat the gas until it expands to a volume of 300.0 m^3 . How much work did the gas do?

4 You have 300.0 m^3 of an ideal gas at $1,500.0 \text{ Pa}$ and heat the gas until it expands to a volume of 900.0 m^3 . How much work did the gas do?

5 You have 50.0 m^3 of an ideal gas at $1,000.0 \text{ Pa}$, and the gas expands, doing $3,000.0 \text{ J}$ of work. What is the final volume of the gas?

6 You have 100.0 m^3 of an ideal gas at 300.0 Pa , and it expands, doing $6,000.0 \text{ J}$ of work. What is the final volume of the gas?

Constant Volume: Isochoric Processes

When an ideal gas's pressure increases at constant volume, how much work is done? Because

$$W = P\Delta V$$

the answer is simple: No work is being done. This is called an *isochoric* process, and you can see a graph of what's happening in Figure 14-3.

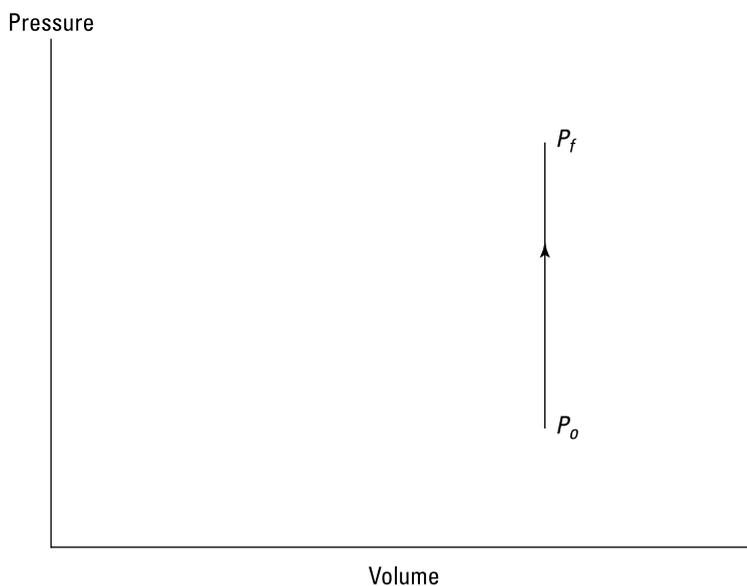


FIGURE 14-3:
An isochoric
graph.

© John Wiley & Sons, Inc.

As you see in the figure, the pressure rises while the volume stays the same, so no work is actually being done.



EXAMPLE

Q. You have 60.0 m^3 of an ideal gas at 200.0 Pa and heat the gas until its pressure is 300.0 Pa at the same volume. How much work did the gas do?

A. The correct answer is none.

1. Use this equation:

$$W = P\Delta V$$

2. Because $\Delta V = 0$, no work was done. This process is an isochoric process.

7 You have 50.0 m^3 of an ideal gas at $1,000.0 \text{ Pa}$ and heat the gas until it has a pressure of $3,000.0 \text{ Pa}$, still at the same volume. How much work did the gas do?

8 You have 300.0 m^3 of an ideal gas at $1,500.0 \text{ Pa}$ and heat the gas until it has a pressure of $6,000.0 \text{ Pa}$. How much work did the gas do?

Constant Temperature: Isothermal Processes

Processes that take place at constant temperature are called *isothermal* processes. What's the work look like in this case as the volume changes? Because $PV = nRT$, the relation between P and V is

$$P = \frac{nRT}{V}$$

That relationship looks like the graph you see in Figure 14-4.

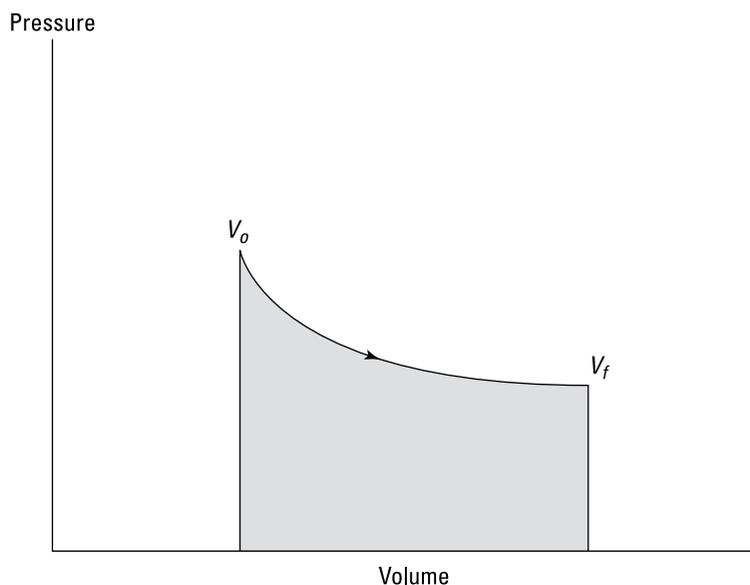


FIGURE 14-4:
An isothermal
graph.

© John Wiley & Sons, Inc.

As before, the work done is the area underneath the graph. What is that area? The work done is given by this equation in this case, where \ln is the natural log:

$$W = nRT \ln(V_f / V_o)$$

Note that because in an isothermal process the temperature stays constant, and because for an ideal gas the internal energy $= (3/2)nRT$, the internal energy doesn't change. So you have

$$\Delta U = 0 = Q - W$$

In other words:

$$Q = W$$

In this case, the work done is equal to the amount that the gas is heated.



Q. You have 1.0 mole of ideal gas at a constant temperature of 20°C , and you expand the gas from $V_o = 0.010 \text{ m}^3$ to $V_f = 0.020 \text{ m}^3$. What work did the gas do in expanding?

EXAMPLE

A. The correct answer is 1,700 J.

1. Use this equation:

$$W = nRT \ln(V_f / V_o)$$

2. Plug in the numbers:

$$W = nRT \ln(V_f / V_o) = (1.0)(8.31 \text{ J/K})(273.15 \text{ K} + 20 \text{ K}) \ln[(0.020 \text{ m}^3) / (0.010 \text{ m}^3)] = 1,690 \text{ J}$$

which rounds to 1,700 J with significant figures.

3. The work done by the gas was 1,700 J. The change in the internal energy of the gas is 0 J, as it must be in isothermal processes. Because $Q = W$, the amount the gas was heated is equal to 1,700 J.

9

You have 1.0 mole of ideal gas at a constant temperature of 30.0°C , and you expand the gas from $V_o = 2.0 \text{ m}^3$ to $V_f = 3.0 \text{ m}^3$. What work did the gas do in expanding?

10

You have 0.60 mole of ideal gas at a constant temperature of 25°C , and you expand the gas from $V_o = 1.0 \text{ m}^3$ to $V_f = 3.0 \text{ m}^3$. What heat was supplied to the gas in expanding?

At Constant Heat: Adiabatic

The last type of thermodynamic process is the *adiabatic* process, in which there is no heat between the system and its surroundings. Take a look at the system in Figure 14-5; everything is surrounded by an insulating substance, so the system can't heat its surroundings. When a change occurs, it's going to be an adiabatic change.

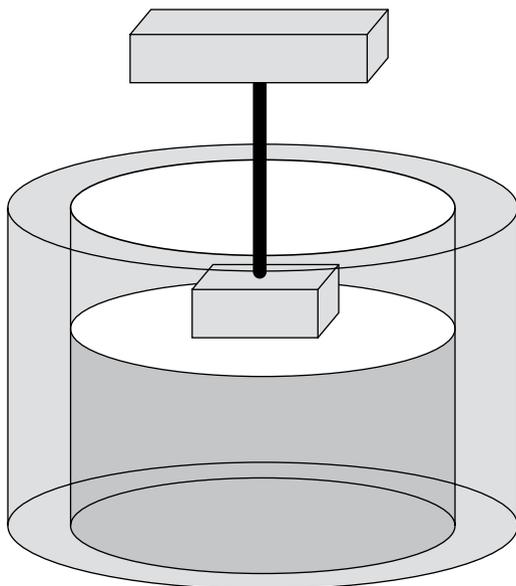


FIGURE 14-5:
An adiabatic system.

© John Wiley & Sons, Inc.

Okay, what's the work done during an adiabatic process? Here, $\Delta Q = 0$, so $\Delta U = -W$. Because the internal energy of an ideal gas is $U = (3/2)nRT$, the work done is

$$W = \frac{3}{2}nR(T_o - T_f)$$

In other words, if the gas does work in an adiabatic process, the work comes from a change in temperature. If the final temperature is lower, the system does work on its surroundings.

You can see what P versus V looks like for an adiabatic process in Figure 14-6. The adiabatic curve here, called an *adiabat*, isn't the same as the isothermal curves, called isotherms. The work done when there is no heat between the system and its surroundings is the area under the adiabat, as shown in Figure 14-6.

You can relate the initial pressure and volume to the final pressure and volume in an adiabatic process this way:

$$P_o V_o^\gamma = P_f V_f^\gamma$$

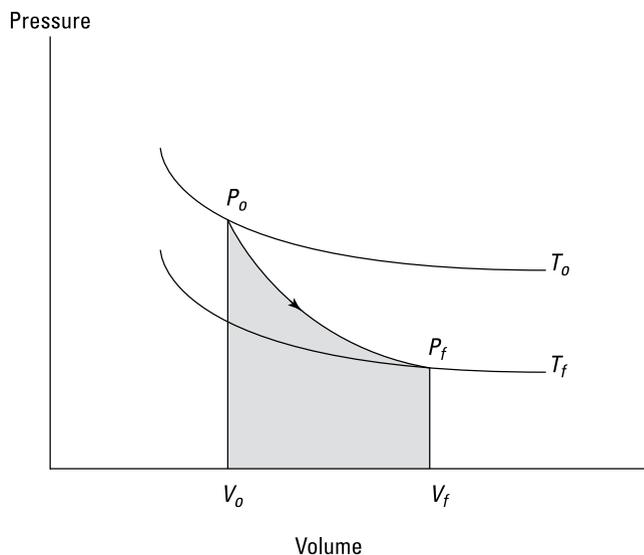


FIGURE 14-6:
An adiabatic
graph.

© John Wiley & Sons, Inc.

What's γ ? It's the ratio of the specific heat capacity of an ideal gas at constant pressure divided by the specific heat capacity of an ideal gas at constant volume, C_p/C_v . What are C_p and C_v ?

$$C_v = \frac{3}{2}R$$

$$C_p = \frac{5}{2}R$$

That makes the ratio γ this:

$$\gamma = \frac{C_p}{C_v}$$

So you can connect pressure and volume at any two points along an adiabat this way:

$$P_o V_o^{5/3} = P_f V_f^{5/3}$$

These processes are adiabatic; there is no heat between the system and its surroundings.



Q. You start with 3.0 L of gas at 1.0 atmosphere and end up, after an adiabatic change, with 6.0 L of gas. What is the new pressure?

EXAMPLE

A. The correct answer is 0.31 atmosphere.

1. Use this equation:

$$P_f = \frac{P_o V_o^{5/3}}{V_f^{5/3}}$$

2. Plug in the numbers:

$$P_f = \frac{P_o V_o^{5/3}}{V_f^{5/3}} = \frac{(1.0 \text{ atm})(3.0 \text{ L})^{5/3}}{(6.0 \text{ L})^{5/3}} = 0.31 \text{ atmosphere}$$

11 You start with 1.0 L of gas at 1.0 atmosphere and end up after an adiabatic change with 3.0 L of gas. What is the new pressure?

12 You start with 1.5 L of gas at 1.7 atmospheres and end up after an adiabatic change with 2.9 L of gas. What is the new pressure?

13 You have 1.0 mole of ideal gas that undergoes an adiabatic change, going from 30.0°C to 10.0°C. What work did the gas do?

14 You have 3.0 moles of ideal gas that undergo an adiabatic change, going from 23°C to 69°C. What work was done on the gas?

The Direction of Heat: The Second Law of Thermodynamics

You may see physics problems related to the definition of the second law of thermodynamics, which says,

If no work is involved, then heat will always be from the object at higher temperature to the object at lower temperature.



EXAMPLE

Q. A red-hot horseshoe is placed on an anvil at room temperature. Which way does the heat go between these two objects?

A. The correct answer is into the anvil.

The second law of thermodynamics tells you that heat is from hotter objects to cooler ones, so the heat is from the red-hot horseshoe to the room-temperature anvil.

Making Heat Work: Heat Engines

You know how a steam engine works: Heated steam does the work. Physics makes a study of this process, and you can see what's going on diagrammatically in Figure 14-7. Heat is supplied to a heat engine, which does work and sends its exhaust to a lower-temperature heat sink. (Often, the heat sink is just the surroundings of the heat engine, as is the case with steam engines.)

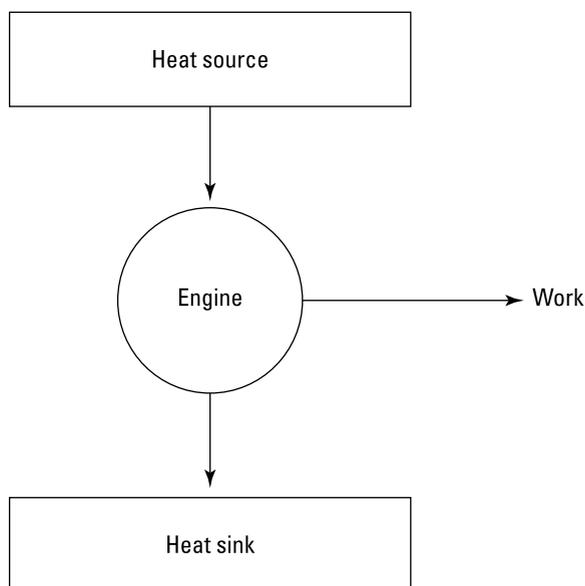


FIGURE 14-7: A heat engine.

© John Wiley & Sons, Inc.

Say that the heat supplied to the engine is Q_h , and the heat sent to the heat sink is Q_c (h and c stand for the hot and cold reservoirs of heat). In that case, you could say that the efficiency of the work engine in terms of turning heat into work is

$$\text{Efficiency} = \frac{\text{Work}}{\text{Heat Input}} = \frac{W}{Q_h}$$

So if all the input heat is converted to work, the efficiency is 1.0. If none of the input heat is converted to work, the efficiency is 0.0.

Note that because total energy is conserved, the heat into the engine must equal the work done plus the heat sent to the heat sink, which means:

$$Q_h = W + Q_c$$

That in turn means you can rewrite the efficiency in terms of just Q_h and Q_c this way:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$



EXAMPLE

Q. Your car is powered by a heat engine and does 3.0×10^7 J of work getting you up a small hill. If the heat engine is 25 percent efficient, how much heat did it use, and how much did it exhaust?

A. The correct answer is 1.2×10^8 J, 0.9×10^8 J.

1. Use this equation:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{3.0 \times 10^7 \text{ J}}{Q_h} = 0.25$$

2. Solve for Q_h :

$$Q_h = \frac{3.0 \times 10^7 \text{ J}}{0.25} = 1.2 \times 10^8 \text{ J}$$

So the input heat was 1.2×10^8 J.

3. Use this equation:

$$Q_h = W + Q_c$$

4. Solve for Q_c :

$$Q_h - W = Q_c$$

5. Plug in the numbers:

$$Q_h - W = 1.2 \times 10^8 \text{ J} - 3.0 \times 10^7 \text{ J} = 0.9 \times 10^8 \text{ J} = Q_c$$

So the output heat was 0.9×10^8 J.

15 A car running on a heat engine does 7.0×10^7 J of work. If the heat engine is 26 percent efficient, how much heat did it use, and how much did it exhaust?

16 A car running using a heat engine does 3.9×10^7 J of work. If the heat engine is 29 percent efficient, how much heat did it use, and how much did it exhaust?

17 A 63 percent efficient heat engine does 3.8×10^{10} J of work. How much heat did it use, and how much did it exhaust?

18 An 87 percent efficient heat engine does 4.5×10^{10} J of work. How much heat did it use, and how much did it exhaust?

19 A heat engine does 4.6×10^7 J of work when supplied 8.9×10^7 J. What is its efficiency?

20 A heat engine does 8.1×10^7 J of work when supplied 10.9×10^7 J. What is its efficiency?

Maximum Efficiency: Carnot Heat Engines

An engineer named Sadi Carnot figured out that the maximum possible efficiency of a heat engine is this:

$$\text{Efficiency} = 1 - \frac{T_c}{T_h}$$

The temperatures T_h and T_c are the temperatures of the heat source and heat sink, respectively, measured in Kelvin.

That efficiency is the best a heat engine can get, assuming that no irreversible losses of energy occur due to friction or other causes. When you have a heat engine that does the best a heat engine can do, you have a Carnot engine, and the preceding equation is the expression for its efficiency.



Q. The heat source for a Carnot engine is at 100°C , and the heat sink is at 20°C . What is the engine's efficiency?

EXAMPLE

A. The correct answer is 21 percent.

1. Use this equation:

$$\text{Efficiency} = 1 - \frac{T_c}{T_h}$$

2. Plug in the numbers:

$$\text{Efficiency} = 1 - \frac{T_c}{T_h} = 1 - \frac{(273.15 \text{ K} + 20 \text{ K})}{(273.15 \text{ K} + 100 \text{ K})} = 21\%$$

21 The heat source for a Carnot engine is at 87°C , and the heat sink is at 49°C . What is the engine's efficiency?

22 The heat source for a Carnot engine is at 67°C , and the heat sink is at 29°C . What is the engine's efficiency?

The Third Law of Thermodynamics

This chapter finishes with the third law of thermodynamics, which says

You cannot reach absolute zero through any process which uses a finite number of steps.

In other words, you can get closer and closer to absolute zero step by step, but you can't actually reach it. Physicists have been able to approach absolute zero until they're just a fraction of a degree away, but no one has been able to reach it.

Answers to Problems about Heat and Work

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 **1,000 J**

1. Use this equation:

$$U_f - U_o = \Delta U = Q - W$$

2. Plug in the numbers:

$$\Delta U = 3,000 \text{ J [heat coming in]} - 2,000 \text{ J [work going out]} = 1,000 \text{ J}$$

2 **800 J**

1. Use this equation:

$$U_f - U_o = \Delta U = Q - W$$

2. Plug in the numbers:

$$\Delta U = 2,500 \text{ J [heat coming in]} - 1,700 \text{ J [work going out]} = 800 \text{ J}$$

3 **250,000 J**

1. Use this equation:

$$W = P\Delta V$$

2. Plug in the numbers:

$$W = P\Delta V = (1,000.0 \text{ Pa})(300.0 \text{ m}^3 - 50.0 \text{ m}^3) = 250,000 \text{ J}$$

4 **$9.0 \times 10^5 \text{ J}$**

1. Use this equation:

$$W = P\Delta V$$

2. Plug in the numbers:

$$W = P\Delta V = (1,500.0 \text{ Pa})(900.0 \text{ m}^3 - 300.0 \text{ m}^3) = 9.0 \times 10^5 \text{ J}$$

5 **53 m^3**

1. Use this equation:

$$W = P\Delta V = P(V_f - V_o)$$

2. Solve for V_f :

$$V_f = \frac{W}{P} + V_o$$

3. Plug in the numbers:

$$V_f = \frac{W}{P} + V_o = \frac{3,000.0 \text{ J}}{1,000.0 \text{ Pa}} + 50.0 \text{ m}^3 = 53 \text{ m}^3$$

6 **120 m³**

1. Use this equation:

$$W = P\Delta V = P(V_f - V_o)$$

2. Solve for V_f :

$$V_f = \frac{W}{P} + V_o$$

3. Plug in the numbers:

$$V_f = \frac{W}{P} + V_o = \frac{6,000.0 \text{ J}}{300.0 \text{ Pa}} + 100.0 \text{ m}^3 = 120 \text{ m}^3$$

7 **0 J**

1. Use this equation:

$$W = P\Delta V = P(V_f - V_o)$$

2. Because no change in volume occurred, this process was isochoric, and no work was done.

8 **0 J**

1. Use this equation:

$$W = P\Delta V = P(V_f - V_o)$$

2. Because no change in volume occurred, this process was isochoric, and no work was done.

9 **1,020 J**

1. Use this equation:

$$W = nRT \ln(V_f/V_o)$$

2. Plug in the numbers:

$$W = nRT \ln(V_f/V_o) = (1.0)(8.31 \text{ J/K})(273.15 \text{ K} + 30.0 \text{ K}) \ln[(3.0 \text{ m}^3)/(2.0 \text{ m}^3)] = 1,020 \text{ J}$$

3. The work done by the gas was 1,020 J. The change in the internal energy of the gas is 0 J, as it must be in isothermal processes. Because $Q = W$, the heat added to the gas is equal to 1,020 J.

10 **1,630 J**

1. Use this equation:

$$W = nRT \ln(V_f/V_o)$$

2. Plug in the numbers:

$$W = nRT \ln(V_f/V_o) = (0.60)(8.31 \text{ J/K})(273.15 \text{ K} + 25 \text{ K}) \ln[(3.0 \text{ m}^3)/(1.0 \text{ m}^3)] = 1,630 \text{ J}$$

3. The work done by the gas was 1,630 J. The change in the internal energy of the gas is 0 J, as it must be in isothermal processes. Because $Q = W$, the heat added to the gas is equal to 1,630 J.

11 0.16 atmosphere

1. Use this equation:

$$P_f = \frac{P_o V_o^{5/3}}{V_f^{5/3}}$$

2. Plug in the numbers:

$$P_f = \frac{P_o V_o^{5/3}}{V_f^{5/3}} = \frac{(1.0 \text{ atm})(1.0 \text{ L})^{5/3}}{(3.0 \text{ L})^{5/3}} = 0.16 \text{ atmosphere}$$

12 0.57 atmosphere

1. Use this equation:

$$P_f = \frac{P_o V_o^{5/3}}{V_f^{5/3}}$$

2. Plug in the numbers:

$$P_f = \frac{P_o V_o^{5/3}}{V_f^{5/3}} = \frac{(1.7 \text{ atm})(1.5 \text{ L})^{5/3}}{(2.9 \text{ L})^{5/3}} = 0.57 \text{ atmosphere}$$

13 250 J

1. Use this equation:

$$W = \frac{3}{2} nR(T_o - T_f)$$

2. Plug in the numbers (a change of 1 K is the same as a change of 1°C):

$$W = \frac{3}{2} nR(T_o - T_f) = (1.5)(1.0)(8.31 \text{ J/}^\circ\text{C})(30^\circ\text{C} - 10^\circ\text{C}) = 250 \text{ J}$$

14 1,720 J

1. Use this equation:

$$W = \frac{3}{2} nR(T_o - T_f)$$

2. Plug in the numbers:

$$W = \frac{3}{2} nR(T_o - T_f) = (1.5)(3.0)(8.31 \text{ J/}^\circ\text{C})(23^\circ\text{C} - 69^\circ\text{C}) = -1,720 \text{ J}$$

3. The work done by the gas is -1,720 J, so the work done on the gas is 1,720 J.

15 $2.7 \times 10^8 \text{ J}$, $2.0 \times 10^8 \text{ J}$

1. Use this equation:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{7.0 \times 10^7 \text{ J}}{Q_h} = 0.26$$

2. Solve for Q_h :

$$Q_h = \frac{7.0 \times 10^7 \text{ J}}{0.26} = 2.7 \times 10^8 \text{ J}$$

So the input heat was $2.7 \times 10^8 \text{ J}$.

3. Use this equation:

$$Q_h = W + Q_c$$

4. Solve for Q_c :

$$Q_h - W = Q_c$$

5. Plug in the numbers:

$$Q_h - W = 2.7 \times 10^8 \text{ J} - 7.0 \times 10^7 \text{ J} = 2.0 \times 10^8 \text{ J} = Q_c$$

So the output heat was $2.0 \times 10^8 \text{ J}$.

16 $1.3 \times 10^8 \text{ J}$, $0.9 \times 10^8 \text{ J}$

1. Use this equation:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{3.9 \times 10^7 \text{ J}}{Q_h} = 0.29$$

2. Solve for Q_h :

$$Q_h = \frac{3.9 \times 10^7 \text{ J}}{0.29} = 1.3 \times 10^8 \text{ J}$$

So the input heat was $1.3 \times 10^8 \text{ J}$.

3. Use this equation:

$$Q_h = W + Q_c$$

4. Solve for Q_c :

$$Q_h - W = Q_c$$

5. Plug in the numbers:

$$Q_h - W = 1.3 \times 10^8 \text{ J} - 3.9 \times 10^7 \text{ J} = 0.9 \times 10^8 \text{ J} = Q_c$$

So the output heat was $0.9 \times 10^8 \text{ J}$.

17 $6.0 \times 10^{10} \text{ J}$, $2.2 \times 10^{10} \text{ J}$

1. Use this equation:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{3.8 \times 10^{10} \text{ J}}{Q_h} = 0.63$$

2. Solve for Q_h :

$$Q_h = \frac{3.8 \times 10^{10} \text{ J}}{0.63} = 6.0 \times 10^{10} \text{ J}$$

So the input heat was 6.0×10^{10} J.

3. Use this equation:

$$Q_h = W + Q_c$$

4. Solve for Q_c :

$$Q_h - W = Q_c$$

5. Plug in the numbers:

$$Q_h - W = 6.0 \times 10^{10} \text{ J} - 3.8 \times 10^{10} \text{ J} = 2.2 \times 10^{10} \text{ J} = Q_c$$

So the output heat was 2.2×10^{10} J.

18 $5.2 \times 10^{10} \text{ J}$, $0.7 \times 10^{10} \text{ J}$

1. Use this equation:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{4.5 \times 10^{10} \text{ J}}{Q_h} = 0.87$$

2. Solve for Q_h :

$$Q_h = \frac{4.5 \times 10^{10} \text{ J}}{0.87} = 5.2 \times 10^{10} \text{ J}$$

So the input heat was 5.2×10^{10} J.

3. Use this equation:

$$Q_h = W + Q_c$$

4. Solve for Q_c :

$$Q_h - W = Q_c$$

5. Plug in the numbers:

$$Q_h - W = 5.2 \times 10^{10} \text{ J} - 4.5 \times 10^{10} \text{ J} = 0.7 \times 10^{10} \text{ J} = Q_c$$

So the output heat was 0.7×10^{10} J.

19 **52 percent**

1. Use this equation:

$$\text{Efficiency} = \frac{W}{Q_h}$$

2. Plug in the numbers:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{4.6 \times 10^7 \text{ J}}{8.9 \times 10^7 \text{ J}} = 52\%$$

20 **74 percent**

1. Use this equation:

$$\text{Efficiency} = \frac{W}{Q_h}$$

2. Plug in the numbers:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{8.1 \times 10^7 \text{ J}}{10.9 \times 10^7 \text{ J}} = 74\%$$

21 **11 percent**

1. Use this equation:

$$\text{Efficiency} = 1 - \frac{T_c}{T_h}$$

2. Plug in the numbers:

$$\text{Efficiency} = 1 - \frac{T_c}{T_h} = 1 - \frac{(273 \text{ K} + 49 \text{ K})}{(273 \text{ K} + 87 \text{ K})} = 11\%$$

22 **11 percent**

1. Use this equation:

$$\text{Efficiency} = 1 - \frac{T_c}{T_h}$$

2. Plug in the numbers:

$$\text{Efficiency} = 1 - \frac{T_c}{T_h} = 1 - \frac{(273 \text{ K} + 29 \text{ K})}{(273 \text{ K} + 67 \text{ K})} = 11\%$$

5

Zap: Electricity

IN THIS PART . . .

Apply physics concepts to static electricity.

See what happens when electrons start moving as an electric current.

IN THIS CHAPTER

- » Checking out electric charges
- » Measuring forces from charges
- » Handling electrical forces as vectors
- » Understanding electric fields
- » Calculating electric fields in parallel plate capacitors

Chapter 15

Static Electricity: Electrons at Rest

This chapter is about electrical charges — the kind that go zap, not the kind that flow in wires. It's all about static electricity here. In this chapter, you see how to solve all kinds of problems, from forces between charges to electric fields to voltage.

Talking about Electric Charges

The unit of charge in the MKS system is the coulomb, symbol C. Electric charges are made up of subatomic charges such as electrons and protons, and a coulomb contains many, many electrons. The charges of subatomic particles can be expressed in terms of the elementary charge, known as e :

$$e = 1.6 \times 10^{-19} \text{ C}$$

Charges come in two types — positive and negative, + and -. For example, each proton has charge $+e = +1.6 \times 10^{-19} \text{ C}$, and each electron has charge $-e = -1.6 \times 10^{-19} \text{ C}$.



Q. How many electrons are in 1.0 coulomb?

A. The correct answer is 6.25×10^{18} electrons.

1. You know that the electric charge of an electron is minus the elementary charge:

$$-e = -1.6 \times 10^{-19} \text{ C}$$

2. To find the total number of electrons in 1.0 coulomb, divide by the elementary charge:

$$\text{Number} = \frac{1.0 \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 6.25 \times 10^{18} \text{ electrons}$$

Notice that in problems like this, 1.0 coulomb refers to the magnitude of the charge, without the sign. Because the electron charge is negative, you know that the actual charge of 6.25×10^{18} electrons, including the sign, is -1.0 C .

1 How many electrons are in 0.60 coulomb?

2 How many electrons are in 800.0 coulombs?

Getting Forceful with Charges

A force exists between electrical charges. Say you have two charges, q_1 and q_2 , and they are a distance r apart. The force pulling them together (if the charges have opposite signs, + and -) or forcing them apart (if the charges have the same sign, + and + or - and -) is

$$F = \frac{kq_1q_2}{r^2}$$

What is k ? You can measure this constant, and its value is $8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$.

Sometimes, you also see this equation written using a constant ϵ_0 , which is called the *permittivity of free space*, like this:

$$F = \frac{q_1q_2}{4\pi\epsilon_0r^2}$$

The constant ϵ_0 has the value $8.854 \times 10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)$.



EXAMPLE

Q. What is the force between 2 coulombs of positive charge 1.0 km apart?

A. The correct answer is 8.99×10^3 N away from each other.

1. Use this equation:

$$F = \frac{kq_1q_2}{r^2}$$

2. Plug in the numbers:

$$F = \frac{kq_1q_2}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.0 \text{ C})(1.0 \text{ C})}{(1,000 \text{ m})^2} = 8.99 \times 10^3 \text{ N}$$

Because the charges have the same sign, the force pushes them apart.

3 What is the force between an electron and a proton 1.0×10^{-8} m apart?

4 What is the force between two protons 1.0 cm apart?

5 What is the force between a charge of 1.0×10^{-3} coulombs and a charge of -3.0×10^{-3} coulombs 1.0 m apart?

6 What is the force between a charge of 1 coulomb and a charge of 2 coulombs, both positive, 10.0 m apart?

Electrical Forces Are Vectors

Take a look at Figure 15-1, which shows three charges: two negative and one positive. The negative charges are at $(-1.0, 0.0)$ and $(1.0, 0.0)$, and the positive charge is at $(0.0, 1.0)$, where all distances are in centimeters. Notice that charges may be labeled either by q , as in the previous example, or by Q , as here.

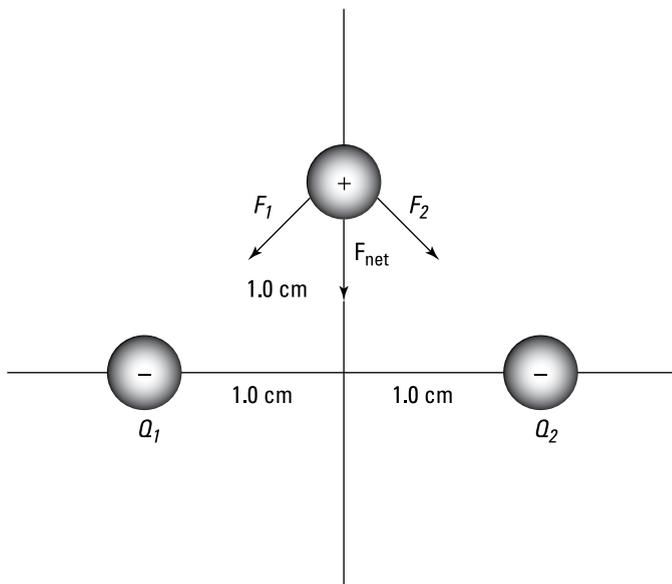


FIGURE 15-1:
Three charges.

© John Wiley & Sons, Inc.

Because the force between electrical charges is a vector, those forces add as vectors. You might try to figure out the net force on the positive charge in the figure, for example.



EXAMPLE

Q. What is the net force on the positive charge if all charges have the magnitude 1.0×10^{-3} coulombs?

A. The correct answer is 6.36×10^7 N downward.

- As you can see in the figure $F_1 = F_2$, each of these two forces is at an angle of 45° from the vertical (because the triangle formed by the positive charge, the center where the axes cross, and either negative charge is a 45-45-90 triangle). So, the downward component of F_1 is $F_1 \cos(45^\circ)$, and the downward component of F_2 has the same value. So you have:

$$F_{net} = 2F_1 \cos(45^\circ) = \sqrt{2}F_1$$

- Solve for F_1 :

$$\begin{aligned} F_1 = \frac{kQ_1Q_2}{r^2} &= \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.0 \times 10^{-3} \text{ C})(1.0 \times 10^{-3} \text{ C})}{[(0.01 \text{ m})^2 + (0.01 \text{ m})^2]} \\ &= 4.5 \times 10^7 \text{ N} \end{aligned}$$

3. So that makes F_{net} :

$$F_{net} = 2F_1 \cos(45^\circ) = \sqrt{2}F_1 = \sqrt{2} (4.5 \times 10^7 \text{ N}) = 6.36 \times 10^7 \text{ N}$$

As you can see in the figure, F_{net} points downward.

7 If the charges in Figure 15-1 all have the magnitude 1.0×10^{-8} C, what is the net force on the positive charge?

8 If the charges in Figure 15-1 are two electrons and a proton, what is the net force on the proton?

Force at a Distance: Electric Fields

The electric field at a particular location is defined as the force felt per coulomb at that location. Here's the equation for electric field (note that it's a vector):

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

What is the electric field from a single item of charge (often called a *point charge* because its physical dimensions are negligible), such as with a single electron? You know that for a point charge Q , the force felt by a charge q distance r away is

$$F = \frac{kqQ}{r^2}$$

What's the electric field from this point charge? Just divide by the magnitude of your test charge, q , to get the force per coulomb:

$$E = \frac{F}{q} = \frac{kQ}{r^2}$$

What are the units? You have force divided by coulombs, so the units are N/C.

An electric field is drawn with lines that show the direction of the electric field vector at all points, as you can see in Figure 15-2. Electric fields point toward negative charges and away from positive charges.

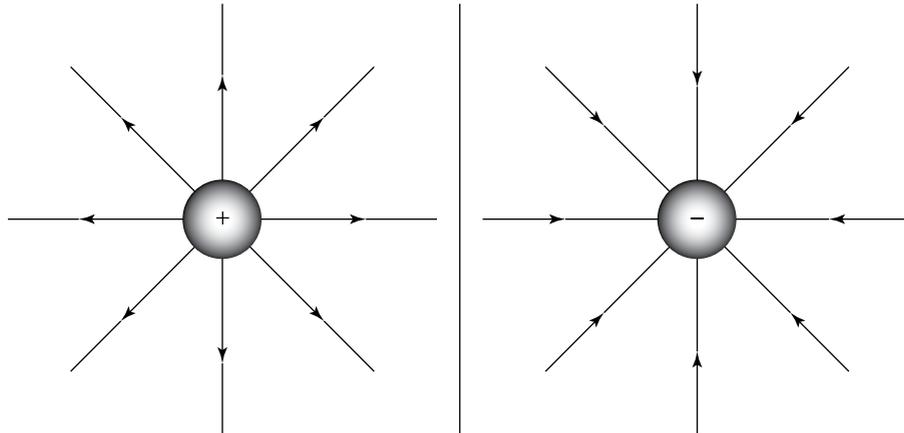


FIGURE 15-2: Electric fields from point charges.

© John Wiley & Sons, Inc.



EXAMPLE

Q. What is the electric field 1.0 cm away from a proton?

A. The correct answer is 1.4×10^{-5} N/C radially outward.

1. Use this equation:

$$E = \frac{kQ}{r^2}$$

2. Plug in the numbers:

$$E = \frac{kQ}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19} \text{ C})}{(0.01 \text{ m})^2} = 1.4 \times 10^{-5} \text{ N/C}$$

The electric field of a positive charge extends radially outward.

9 What is the electric field from an electron 1.0 cm away from it?

10 What is the electric field from a point charge of -1.0 C 1.0 m away from it?

Easy Electric Field: Parallel Plate Capacitors

The electric field caused by point charges is a little difficult to work with because it's not constant in space. That's why you often see problems involving parallel plate capacitors, as shown in Figure 15-3, because between the plates of a parallel plate capacitor the electric field is indeed constant. Parallel plate capacitors are typically made of metal plates, each of which holds a charge — one plate positive and the other negative. Capacitors like this can store separated charge, and the electric field between them is constant.

$$E = \frac{q}{\epsilon_0 A}$$

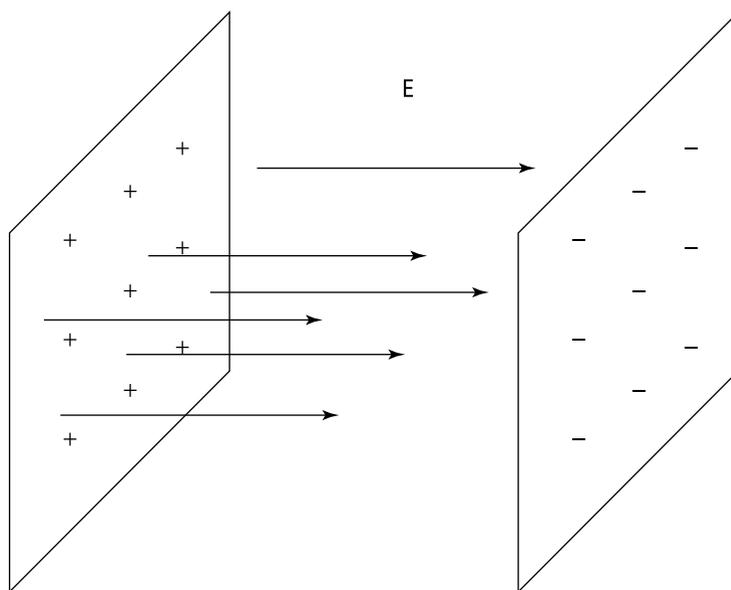


FIGURE 15-3:
A parallel plate
capacitor.

© John Wiley & Sons, Inc.

Where ϵ_0 is the permittivity of free space, $8.854 \times 10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)$, q is the total charge on either of the plates (one plate has charge $+q$ and the other $-q$), and A is the area of each plate. This is also often written in terms of the charge density, σ , on each plate, where $\sigma = q/A$ (the charge per square meter), and here's what charge density makes this equation look like:

$$E = \frac{q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0}$$

This electric field is constant between the plates and points from the positive plate toward the negative one.



EXAMPLE

Q. You have a parallel plate capacitor with plates of 1.0 m^2 , and the magnitude of charge on each plate is 1.0 C . What is the electric field between the plates?

A. The correct answer is $1.1 \times 10^{11} \text{ N/C}$ from the positive plate toward the negative one.

1. Use this equation:

$$E = \frac{q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0}$$

2. Plug in the numbers:

$$\begin{aligned} E = \frac{q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0} &= \frac{1.0 \text{ C}}{(8.854 \times 10^{-12} \text{ C}^2 / \text{N}\cdot\text{m}^2)(1.0 \text{ m}^2)} \\ &= 1.1 \times 10^{11} \text{ N/C} \end{aligned}$$

The electric field points from the positive plate toward the negative one.

11 You have a parallel plate capacitor with plates of 1.5 m^2 , and the magnitude of charge on each plate is 2.0 C . What is the electric field between the plates?

12 You have a parallel plate capacitor with plates of 2.3 m^2 , and the magnitude of charge on each plate is 0.75 C . What is the electric field between the plates?

13 You have a parallel plate capacitor with plates of 1.0 m^2 , and the magnitude of charge on each plate is 2.0 C . What is the force on an electron between the plates?

14 You have a parallel plate capacitor with plates of 1.3 m^2 , and the magnitude of charge on each plate is 6.7 C . What is the force on an electron between the plates?

Ramping Up Some Voltage

Electric potential, called *voltage* (and, as you already know, measured in volts), is the work needed to move a charge from the negative side of an electric field to the positive side, divided by the amount of the charge you're moving. The electric potential is the potential energy gained per coulomb by moving from the negative side to the positive side.

The work to move a charge q from the negative plate a distance s toward the positive plate in a parallel plate capacitor is

$$W = Fs$$

And you know that $F = qE$, so:

$$W = Fs = qEs$$

That work becomes the charge's potential energy, so the electric potential at that location is

$$V = \frac{PE}{q} = \frac{W}{q} = Es$$

For a parallel plate capacitor, you already know:

$$E = \frac{q}{\epsilon_0 A}$$

And the voltage between the plates separated by a distance s is

$$V = Es$$

So:

$$V = \frac{qs}{\epsilon_0 A}$$

That's the voltage between two plates of area A and charge q , separation s .



Q. You have a parallel plate capacitor with plates of 1 m^2 , $1.0 \times 10^{-2} \text{ m}$ apart, and the magnitude of charge on each plate is $1.0 \times 10^{-6} \text{ C}$. What is the voltage between the plates?

EXAMPLE

A. The correct answer is 1.1×10^3 volts.

1. Use this equation:

$$V = \frac{qs}{\epsilon_0 A}$$

2. Plug in the numbers:

$$\begin{aligned} V &= \frac{qs}{\epsilon_0 A} = \frac{(1.0 \times 10^{-6} \text{ C})(1.0 \times 10^{-2} \text{ m})}{(8.854 \times 10^{-12} \text{ C}^2 / \text{N}\cdot\text{m}^2)(1.0 \text{ m}^2)} \\ &= 1.1 \times 10^3 \text{ volts} \end{aligned}$$

15 You have a parallel plate capacitor with plates of 1.0 m^2 , 0.10 m apart, and the magnitude of charge on each plate is $6.0 \times 10^{-7} \text{ C}$. What is the voltage between the plates?

16 You have a parallel plate capacitor with plates of 2.0 m^2 , 3.0 cm apart, and the magnitude of charge on each plate is $1.0 \times 10^{-8} \text{ C}$. What is the voltage between the plates?

17 You have a parallel plate capacitor with plates of 2.0 m^2 , 2.0 cm apart, and the magnitude of charge on each plate is $2.0 \times 10^{-7} \text{ C}$. How much work does it take to move $1.0 \times 10^{-8} \text{ C}$ of charge from the negative to the positive plate?

18 You have a parallel plate capacitor with plates of 1.0 m^2 , 1.3 cm apart, and the magnitude of charge on each plate is $1.5 \times 10^{-6} \text{ C}$. How much work does it take to move $3.0 \times 10^{-8} \text{ C}$ of charge from the negative to the positive plate?

Electric Potential from Point Charges

What's the potential some distance away from a test charge? You know that in general:

$$V = Es$$

For a point charge Q , however, the electric field isn't constant as it is between the plates of a capacitor. You know that the force on a test charge q from point charge Q is equal to:

$$F = \frac{kQq}{r^2}$$

And you know that the electric field around a point charge Q is equal to:

$$E = \frac{kQ}{r^2}$$

At an infinite distance away from the point charge, the potential will be zero. As you bring a test charge closer, to a point r away from the point charge, you have to add up all the work you do and then divide by the size of the test charge. That ends up looking like this:

$$V = \frac{kQ}{r}$$

That equation is the electric potential, measured in volts, at any point a distance r from a point charge of charge Q , where zero potential is at $r = \infty$. It's the work needed to bring a test charge from ∞ to the distance r from the point charge, divided by the magnitude of the test charge.



Q. What is the electric potential 1.0 m from a point charge of 1.0 C, assuming that the potential is zero at $r = \infty$?

EXAMPLE

A. The correct answer is 8.99×10^9 volts.

1. Use this equation:

$$V = \frac{kQ}{r}$$

2. Plug in the numbers:

$$V = \frac{kQ}{r} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2)(1.0 \text{ C})}{(1.0 \text{ m})} = 8.99 \times 10^9 \text{ volts}$$

19 What is the electric potential 100.0 m from a point charge of 0.10 C, assuming that the potential is zero at $r = \infty$?

20 What is the electric potential 12.0 m from a point charge of 0.40 C, assuming that the potential is zero at $r = \infty$?

21 How much work does it take to bring a 0.010 C charge from ∞ to 1.0 m from a 5.0 C charge?

22 How much work does it take to bring a 0.020 C charge from ∞ to 4.0 m from a 10.0 C charge?

Answers to Problems about Static Electricity

The following are the answers to the practice questions presented earlier in this chapter. You see how to work out each answer, step by step.

1 3.75×10^{18} electrons

1. Use this equation:

$$e = 1.6 \times 10^{-19} \text{ C}$$

2. Plug in the numbers:

$$\text{Number} = \frac{0.60 \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 3.75 \times 10^{18} \text{ electrons}$$

2 5.0×10^{21} electrons

1. Use this equation:

$$e = 1.6 \times 10^{-19} \text{ C}$$

2. Plug in the numbers:

$$\text{Number} = \frac{800.0 \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 5.0 \times 10^{21} \text{ electrons}$$

3 2.3×10^{-12} N pulling them together

1. Use this equation:

$$F = \frac{kq_1q_2}{r^2}$$

2. Plug in the numbers:

$$F = \frac{kq_1q_2}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2)(1.6 \times 10^{-19} \text{ C})(1.6 \times 10^{-19} \text{ C})}{(1.0 \times 10^{-8} \text{ m})^2} = 2.3 \times 10^{-12} \text{ N}$$

Because the charges have opposite signs, the force pulls them together.

4 2.3×10^{-24} N pushing them apart

1. Use this equation:

$$F = \frac{kq_1q_2}{r^2}$$

2. Plug in the numbers:

$$F = \frac{kq_1q_2}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2)(1.6 \times 10^{-19} \text{ C})(1.6 \times 10^{-19} \text{ C})}{(1.0 \times 10^{-2} \text{ m})^2} = 2.3 \times 10^{-24} \text{ N}$$

Because the charges have the same signs, the force pushes them apart.

5 **27,000 N pulling them together**

1. Use this equation:

$$F = \frac{kq_1q_2}{r^2}$$

2. Plug in the numbers:

$$F = \frac{kq_1q_2}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2)(1.0 \times 10^{-3} \text{ C})(-3.0 \times 10^{-3} \text{ C})}{(1.0 \text{ m})^2} = -27,000 \text{ N}$$

Because the charges have opposite signs, the force pulls them together.

6 **1.8×10^8 N forcing the charges apart**

1. Use this equation:

$$F = \frac{kq_1q_2}{r^2}$$

2. Plug in the numbers:

$$F = \frac{kq_1q_2}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2)(1.0 \text{ C})(2.0 \text{ C})}{(10.0 \text{ m})^2} = 1.8 \times 10^8 \text{ N}$$

Because the charges have the same signs, the force pushes them apart.

7 **6.36×10^{-3} N downward**

1. Add vectors:

$$F_{net} = 2F_1 \cos(45^\circ) = \sqrt{2}F_1$$

2. Solve for F_1 :

$$F_1 = \frac{kQ_1Q_2}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2)(1.0 \times 10^{-8} \text{ C})(1.0 \times 10^{-8} \text{ C})}{[(0.01 \text{ m})^2 + (0.01 \text{ m})^2]} = 4.50 \times 10^{-3} \text{ N}$$

Because the charges have the opposite signs, the force pulls them together.

3. Solve for F_{net} :

$$F_{net} = 2F_1 \cos(45^\circ) = \sqrt{2}F_1 = \sqrt{2} (4.50 \times 10^{-3} \text{ N}) = 6.36 \times 10^{-3} \text{ N}$$

The vector sum of the forces is downward.

8 **1.63×10^{-24} N downward**

1. Add vectors:

$$F_{net} = 2F_1 \cos(45^\circ) = \sqrt{2}F_1$$

2. Solve for F_1 :

$$F_1 = \frac{kQ_1Q}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2)(1.6 \times 10^{-19} \text{ C})(1.6 \times 10^{-19} \text{ C})}{[(0.01 \text{ m})^2 + (0.01 \text{ m})^2]} = 1.15 \times 10^{-24} \text{ N}$$

Because the charges have the opposite signs, the force pulls them together.

3. Solve for F_{net} :

$$F_{net} = 2F_1 \cos(45^\circ) = \sqrt{2}F_1 = \sqrt{2} (1.15 \times 10^{-24} \text{ N}) = 1.63 \times 10^{-24} \text{ N}$$

The vector sum of the forces is downward.

9 Magnitude $1.4 \times 10^{-5} \text{ N/C}$, pointing toward the charge

1. Use this equation:

$$E = \frac{kQ}{r^2}$$

2. Solve for E :

$$E = \frac{kQ}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2)(-1.6 \times 10^{-19} \text{ C})}{(0.01 \text{ m})^2} = -1.4 \times 10^{-5} \text{ N/C}$$

The electric field of a negative charge points radially inward, toward the charge.

10 Magnitude $8.99 \times 10^9 \text{ N/C}$, pointing toward the charge

1. Use this equation:

$$E = \frac{kQ}{r^2}$$

2. Solve for E :

$$E = \frac{kQ}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2)(-1.0 \text{ C})}{(1.0 \text{ m})^2} = -8.99 \times 10^9 \text{ N/C}$$

The electric field of a negative charge points radially inward, toward the charge.

11 Magnitude $1.5 \times 10^{11} \text{ N/C}$, from the positive plate toward the negative one

1. Use this equation:

$$E = \frac{q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0}$$

2. Plug in the numbers:

$$E = \frac{q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0} = \frac{2.0 \text{ C}}{(8.854 \times 10^{-12} \text{ C}^2 / \text{N}\cdot\text{m}^2)(1.5 \text{ m}^2)} = 1.5 \times 10^{11} \text{ N/C}$$

The electric field points from the positive plate toward the negative one.

12 Magnitude 3.7×10^{10} N/C, from the positive plate toward the negative one

1. Use this equation:

$$E = \frac{q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0}$$

2. Plug in the numbers:

$$E = \frac{q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0} = \frac{0.75 \text{ C}}{(8.854 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(2.3 \text{ m}^2)} = 3.7 \times 10^{10} \text{ N/C}$$

The electric field points from the positive plate toward the negative one.

13 Magnitude 3.5×10^{-8} N, toward the positive plate

1. Use this equation:

$$E = \frac{q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0}$$

2. Plug in the numbers:

$$E = \frac{q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0} = \frac{2.0 \text{ C}}{(8.854 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(1.0 \text{ m}^2)} = 2.2 \times 10^{11} \text{ N/C}$$

3. Use this equation to find the force on the electron:

$$F = qE$$

4. Plug in the numbers:

$$F = qE = (1.6 \times 10^{-19} \text{ C})(2.2 \times 10^{11} \text{ N/C}) = 3.5 \times 10^{-8} \text{ N}$$

The force on the negatively charged electron points toward the positive plate.

14 Magnitude 9.32×10^{-8} N, toward the positive plate

1. Use this equation:

$$E = \frac{q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0}$$

2. Plug in the numbers:

$$E = \frac{q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0} = \frac{6.7 \text{ C}}{(8.854 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(1.3 \text{ m}^2)} = 5.8 \times 10^{11} \text{ N/C}$$

3. Use this equation to find the force on the electron:

$$F = qE$$

4. Plug in the numbers:

$$F = qE = (1.6 \times 10^{-19} \text{ C})(5.8 \times 10^{11} \text{ N/C}) = 9.32 \times 10^{-8} \text{ N}$$

The force on the negatively charged electron points toward the positive plate.

15 6.8×10^3 volts

1. Use this equation:

$$V = \frac{qs}{\epsilon_0 A}$$

2. Plug in the numbers:

$$V = \frac{qs}{\epsilon_0 A} = \frac{(6.0 \times 10^{-7} \text{ C})(0.10 \text{ m})}{(8.854 \times 10^{-12} \text{ C}^2 / \text{N}\cdot\text{m}^2)(1.0 \text{ m}^2)} = 6.8 \times 10^3 \text{ volts}$$

16 **17 volts**

1. Use this equation:

$$V = \frac{qs}{\epsilon_0 A}$$

2. Plug in the numbers to find the voltage between the plates:

$$V = \frac{qs}{\epsilon_0 A} = \frac{(1.0 \times 10^{-8} \text{ C})(3.0 \times 10^{-2} \text{ m})}{(8.854 \times 10^{-12} \text{ C}^2 / \text{N}\cdot\text{m}^2)(2.0 \text{ m}^2)} = 17 \text{ volts}$$

17 $2.3 \times 10^{-6} \text{ J}$

1. Use this equation:

$$V = \frac{qs}{\epsilon_0 A}$$

2. Plug in the numbers to find the voltage between the plates:

$$V = \frac{qs}{\epsilon_0 A} = \frac{(2.0 \times 10^{-7} \text{ C})(2.0 \times 10^{-2} \text{ m})}{(8.854 \times 10^{-12} \text{ C}^2 / \text{N}\cdot\text{m}^2)(2.0 \text{ m}^2)} = 2.3 \times 10^2 \text{ volts}$$

3. Use this equation:

$$V = \frac{W}{q}$$

4. Solve for W :

$$W = qV$$

5. Plug in the numbers:

$$W = qV = (1.0 \times 10^{-8} \text{ C})(2.3 \times 10^2 \text{ V}) = 2.3 \times 10^{-6} \text{ J}$$

18 $6.6 \times 10^{-4} \text{ J}$

1. Use this equation:

$$V = \frac{qs}{\epsilon_0 A}$$

2. Plug in the numbers to find the voltage between the plates:

$$V = \frac{qs}{\epsilon_0 A} = \frac{(1.5 \times 10^{-6} \text{ C})(1.3 \times 10^2 \text{ m})}{(8.854 \times 10^{-12} \text{ C}^2 / \text{N}\cdot\text{m}^2)(1.0 \text{ m}^2)} = 2.2 \times 10^4 \text{ volts}$$

3. Use this equation:

$$V = \frac{W}{q}$$

4. Solve for W :

$$W = qV$$

5. Plug in the numbers:

$$W = qV = (3.0 \times 10^{-8} \text{ C})(2.2 \times 10^4 \text{ V}) = 6.6 \times 10^{-4} \text{ J}$$

19 8.99×10^6 volts

1. Use this equation:

$$V = \frac{kQ}{r}$$

2. Plug in the numbers:

$$V = \frac{kQ}{r} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2)(0.10 \text{ C})}{(100.0 \text{ m})} = 8.99 \times 10^6 \text{ volts}$$

20 3.0×10^8 volts

1. Use this equation:

$$V = \frac{kQ}{r}$$

2. Plug in the numbers:

$$V = \frac{kQ}{r} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2)(0.40 \text{ C})}{(12.0 \text{ m})} = 3.0 \times 10^8 \text{ volts}$$

21 4.5×10^8 J

1. Use this equation:

$$V = \frac{kQ}{r}$$

2. Plug in the numbers:

$$V = \frac{kQ}{r} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2)(5.0 \text{ C})}{(1.0 \text{ m})} = 4.5 \times 10^{10} \text{ volts}$$

3. Use this equation:

$$V = \frac{W}{q}$$

4. Solve for W :

$$W = qV$$

5. Plug in the numbers:

$$W = qV = (0.010 \text{ C})(4.5 \times 10^{10} \text{ V}) = 4.5 \times 10^8 \text{ J}$$

22 $4.5 \times 10^8 \text{ J}$

1. Use this equation:

$$V = \frac{kQ}{r}$$

2. Plug in the numbers:

$$V = \frac{kQ}{r} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2 / \text{C}^2)(10.0 \text{ C})}{(4.0 \text{ m})} = 2.25 \times 10^{10} \text{ volts}$$

3. Use this equation:

$$V = \frac{W}{q}$$

4. Solve for W :

$$W = qV$$

5. Plug in the numbers:

$$W = qV = (0.020 \text{ C})(2.25 \times 10^{10} \text{ V}) = 4.5 \times 10^8 \text{ J}$$

- » Getting to know current
- » Handling resistance
- » Sizing up power
- » Developing series circuits
- » Creating parallel circuits

Chapter 16

Electrons in Motion: Circuits

This chapter is about what happens when you get electrons moving: You get electric current. Electric current runs through wires and has to contend with various items in those wires — resistors, capacitors, and the like. In this chapter, you see how to handle such items yourself.

Electrons in a Whirl: Current

If you have electrons moving through a wire, you have electrical current. *Current*, symbol I , is defined as the amount of charge that passes by in 1 second. One coulomb per second is named 1 ampere or just 1 amp, symbol A. So this equation is how you find current:

$$I = \frac{q}{t}$$

Current is created when you have an *electromotive force* (EMF) providing a voltage across a conductor. EMF can be supplied by batteries, generators, and similar items in physics problems. Notice that EMF, despite its name, isn't actually a force. It's a change in energy per charge, measured in volts.



EXAMPLE

Q. How many electrons per second pass by a given point in a wire carrying 2.0 amps?

A. 1.25×10^{19} electrons

1. Use this equation:

$$I = \frac{q}{t}$$

2. Solve for q :

$$q = It$$

3. Plug in the numbers to find the total charge in 1 second:

$$q = (2.0 \text{ A})(1.0 \text{ s}) = 2.0 \text{ C}$$

4. Divide by the charge per electron to find the number of electrons:

$$\text{Number} = \frac{2.0 \text{ C}}{1.6 \times 10^{-19}} = 1.25 \times 10^{19}$$

1 How many electrons per second pass by a given point in a wire carrying 6.0 amps?

2 What total charge passes by a given point in a circuit carrying 12.0 amps in 1 minute?

Giving You Some Resistance: Ohm's Law

When you have an EMF providing voltage across a wire, how much current is there? The answer turns out to depend on how much resistance, symbol R , is in the wire. This equation shows the relationship among voltage, current, and resistance:

$$V = IR$$

Resistance is measured in ohms, symbol Ω . As you can see from the equation, 1 ohm is 1 volt per amp. In other words, a wire with a resistance of 1 ohm requires 1 V across it to produce a current of 1 A in that wire.

At the top of Figure 16-1, you see a battery with an EMF of 6.0 volts. The current is from the positive terminal of the battery — corresponding to the longer of the two lines in the battery symbol — to the negative terminal of the battery.

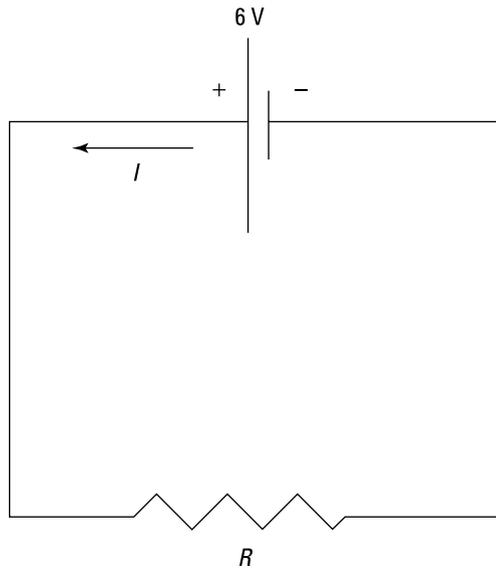


FIGURE 16-1: A resistor and a battery in a circuit.

© John Wiley & Sons, Inc.

At the bottom of the figure is a resistor, R . How much current is there through the resistor? You know the answer from the equation $V = IR$.



Q. If the resistor in Figure 16-1 is 3.0Ω , what is the current through it, driven by the 6.0 V battery?

EXAMPLE

A. 2.0 amps

1. Use this equation:

$$V = IR$$

2. Solve for I :

$$I = \frac{V}{R}$$

3. Plug in the numbers:

$$I = \frac{V}{R} = \frac{6.0 \text{ V}}{3.0 \Omega} = 2.0 \text{ A}$$

3 If the resistor in Figure 16-1 is $1.5\ \Omega$, what is the current through it, driven by the $6.0\ \text{V}$ battery?

4 If the resistor in Figure 16-1 is $0.5\ \Omega$, what is the current through it, driven by the $6.0\ \text{V}$ battery?

5 What's the voltage across a $3.0\ \Omega$ resistor with a current of $1.0\ \text{A}$ going through it?

6 What's the voltage across a $5.0\ \Omega$ resistor with a current of $1.5\ \text{A}$ going through it?

Powering It Up

Notice how light bulbs get hot? That happens because the filament in the light bulb acts as a resistor — and a resistor heats its surroundings. How much does a resistor heat its surroundings? This equation shows how to calculate the answer in terms of current or voltage:

$$P = IV = \frac{V^2}{R} = I^2R$$



EXAMPLE

Q. If a $300.0\ \Omega$ resistor has a current of 1.0 amps going through it, how much power does it turn into heat?

A. 300 watts

1. Use this equation:

$$P = I^2R$$

2. Plug in the numbers:

$$P = I^2R = (1.0\ \text{A})^2(300.0\ \Omega) = 300\ \text{watts}$$

7 If a $500\ \Omega$ resistor has a current of 2.0 amps going through it, how much power does it turn into heat?

8 If a $2,000\ \Omega$ resistor has a current of 1.5 amps going through it, how much power does it turn into heat?

One after the Other: Series Circuits

What if you have several resistors, one after another, in a circuit, as you see in Figure 16-2?

When you have resistors arranged this way, so the current has to go through both of them, the arrangement is called *placing resistors in series*. To find the net resistance, you just add the two individual resistances:

$$R = R_1 + R_2$$

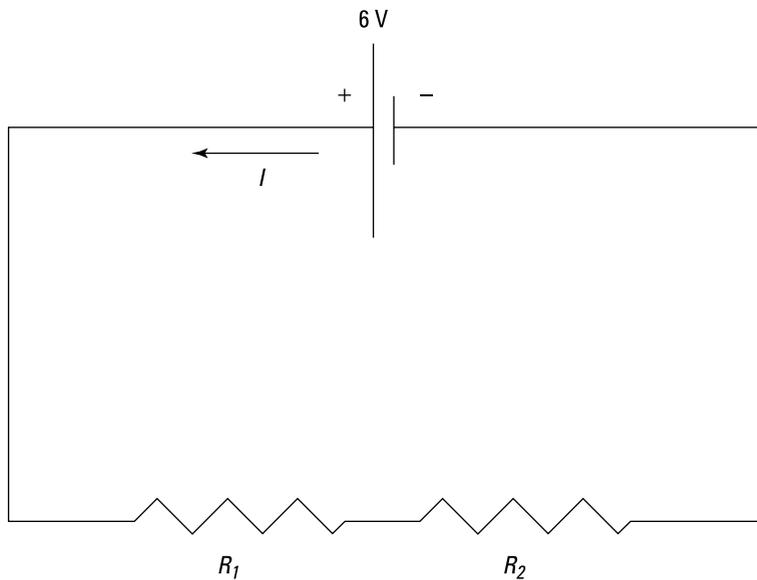


FIGURE 16-2:
Resistors in series.

© John Wiley & Sons, Inc.



Q. If R_1 is $20\ \Omega$, and R_2 is $40\ \Omega$, what is the current flowing in Figure 16-2, in which the battery is $6.0\ \text{V}$?

EXAMPLE

A. $0.1\ \text{A}$

1. Use this equation:

$$R = R_1 + R_2$$

2. Plug in the numbers:

$$R = R_1 + R_2 = 20\ \Omega + 40\ \Omega = 60\ \Omega$$

3. Use this equation to find the current:

$$V = IR$$

4. Solve for I :

$$I = \frac{V}{R}$$

5. Plug in the numbers:

$$I = \frac{V}{R} = \frac{6.0\ \text{V}}{60\ \Omega} = 0.1\ \text{A}$$

9 If R_1 is $60\ \Omega$, and R_2 is $100\ \Omega$, what is the current in Figure 16-2, in which the battery is $6.0\ \text{V}$?

10 If R_1 is $100\ \Omega$, and R_2 is $200\ \Omega$, what is the current in Figure 16-2, in which the battery is $6.0\ \text{V}$?

11 If R_1 is $100\ \Omega$, and R_2 is $200\ \Omega$, what is the voltage across R_1 in Figure 16-2, in which the battery is $6.0\ \text{V}$?

12 If R_1 is $100\ \Omega$, and R_2 is $200\ \Omega$, what is the voltage across R_2 in Figure 16-2, in which the battery is $6.0\ \text{V}$?

All for One: Parallel Circuits

Say you have resistors connected as you see in Figure 16-3 so the current can go through either resistor.

This arrangement is called *placing resistors in parallel*. Unlike resistors in series, in which the current has to go through the resistors one after another, in this arrangement part of the current goes through one resistor and part through the other.

What's the total resistance of two resistors in parallel? This equation shows you:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

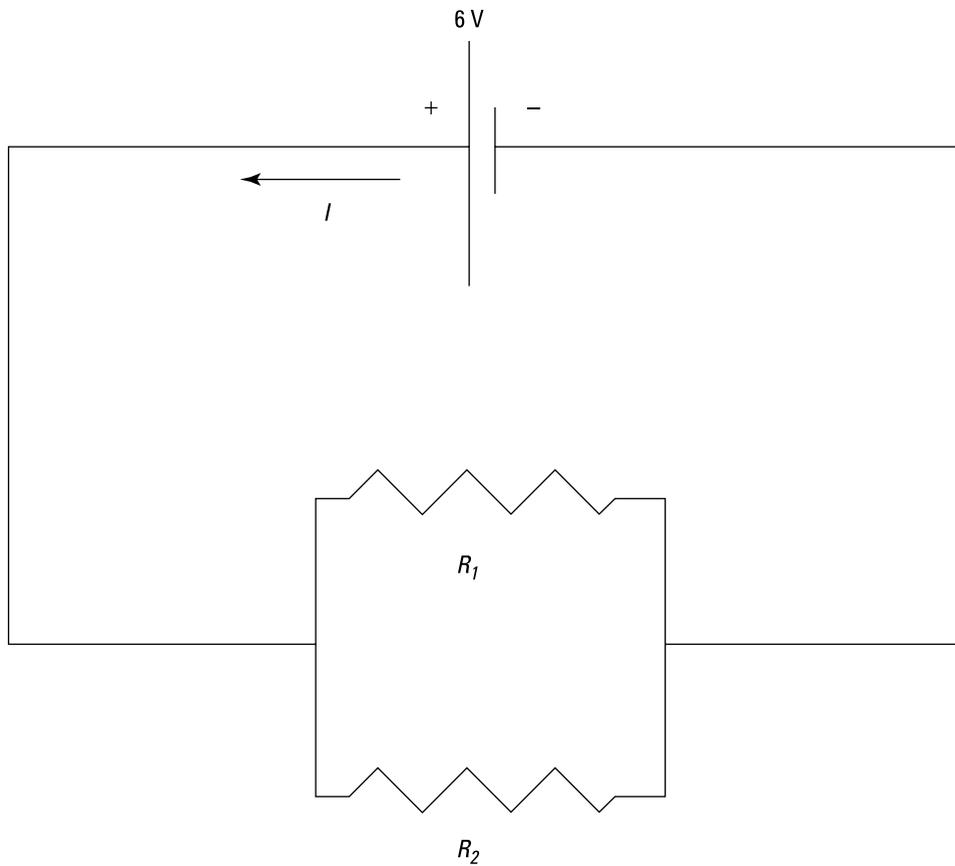


FIGURE 16-3:
Resistors in
parallel.

© John Wiley & Sons, Inc.



Q. If R_1 is $20\ \Omega$, and R_2 is $60\ \Omega$, what is the current in Figure 16-3, in which the battery is $6.0\ \text{V}$?

EXAMPLE

A. $0.4\ \text{A}$

1. Use this equation:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

2. Plug in the numbers:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{20\ \Omega} + \frac{1}{60\ \Omega} = 0.066 / \Omega$$

3. Find R , the total resistance:

$$R = \frac{1}{0.066 / \Omega} = 15\ \Omega$$

4. Use this equation:

$$V = IR$$

5. Solve for I :

$$I = \frac{V}{R}$$

6. Plug in the numbers:

$$I = \frac{V}{R} = \frac{6.0 \text{ V}}{15 \Omega} = 0.4 \text{ A}$$

13 If R_1 is 30Ω , and R_2 is 90Ω , what is the current through the battery in Figure 16-3, in which the battery is 6.0 V ?

14 If R_1 is 45Ω , and R_2 is 120Ω , what is the current through the battery in Figure 16-3, in which the battery is 6.0 V ?

15 If R_1 is 39Ω , and R_2 is 93Ω , what is the current through the battery in Figure 16-3, in which the battery is 6.0 V ?

16 If R_1 is 42Ω , and R_2 is 56Ω , what is the current through the battery in Figure 16-3, in which the battery is 6.0 V ?

The Whole Story: Kirchhoff's Rules

The circuit in Figure 16-4 is a whopper. What's I_1 ? What's I_2 ? Kirchhoff's rules make everything clear.

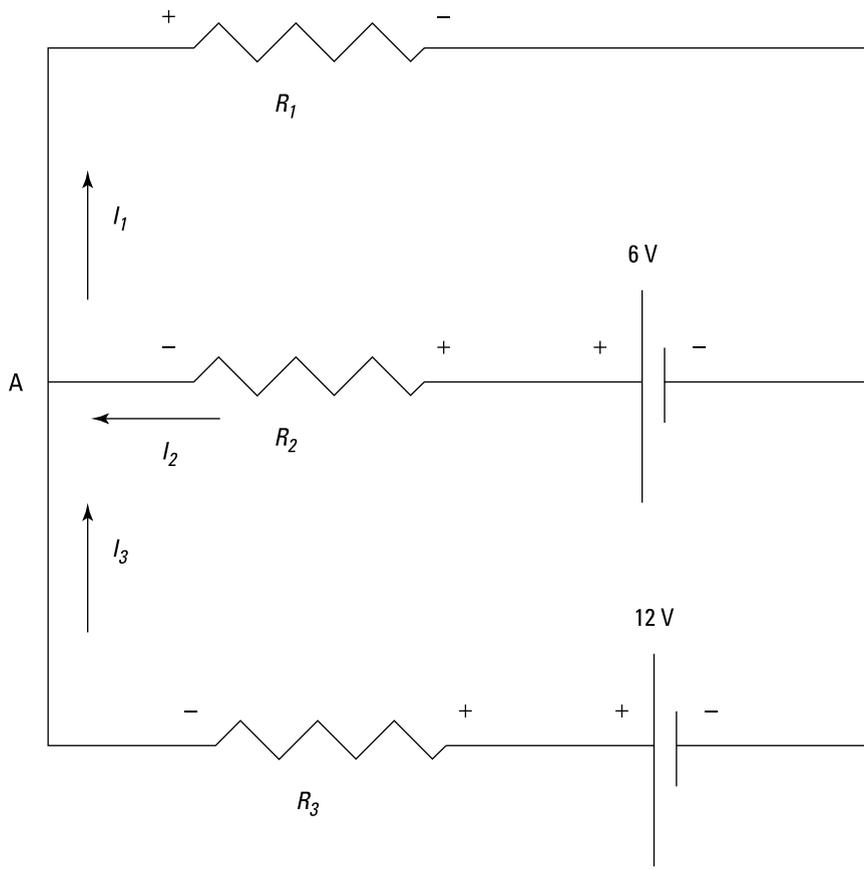


FIGURE 16-4:
A circuit.

© John Wiley & Sons, Inc.

Following are Kirchhoff's rules for a circuit:

- » Junction rule. The junction rule says that the total current going into any point in a circuit must equal the total current going out of that point. So $\Sigma I_{in} = \Sigma I_{out}$ at any point.
- » Loop rule. The loop rule says that around any closed loop in a circuit, the sum of potential rises (as from a battery) must equal the sum of the potential drops (as from resistors). So $\Sigma V = 0$ around any loop.

An easy way of summing the voltages around a loop is to draw in the current arrows (it's no problem if you draw a current arrow backward; the current will just come out negative) and label each resistor with a + where the current goes in and a - where it comes out. Then go around the loop (either direction is fine), adding a $+V$ if you encounter a battery's positive terminal first or a $-V$ if you encounter its negative terminal first, and a $+IR$ or $-IR$ for each

resistor, depending on whether you encounter the + or – signs you added to each resistor first. Then set that whole expression to zero, such as $-6 V + IR_1 + 12 V + IR_2 = 0$.



Q. If R_1 is 2.0Ω , R_2 is 4.0Ω , and R_3 is 6.0Ω , what are the three currents in Figure 16-4, in which the batteries are $6.0 V$ and $12.0 V$?

EXAMPLE

A. $I_1 = 1.91$ amps, $I_2 = 0.55$ amp, $I_3 = 1.36$ amps

- The junction rule says that $\Sigma I_{in} = \Sigma I_{out}$ at any point, so use point A at left in the figure. I_2 and I_3 flow into it, and I_1 flows out of it, so:

$$I_2 + I_3 = I_1$$

- The loop rule says that $\Sigma V = 0$. Figure 16-4 has three loops: the two internal loops and the external overall loop. Because you have three unknowns — I_1 , I_2 , and I_3 — all you need are three equations, and the $\Sigma I_{in} = \Sigma I_{out}$ rule has already given you one. So go around the two internal loops to get two more equations. From the top loop, you get

$$+6 V - (4 \Omega)I_2 - (2 \Omega)I_1 = 0$$

If you divide everything by 1Ω you get

$$+6 A - 4I_2 - 2I_1 = 0$$

- From the bottom loop, you get

$$+12 V - (6 \Omega)I_3 + (4 \Omega)I_2 - 6 V = 0$$

If you divide everything by 1Ω you get

$$+12 A - 6I_3 + 4I_2 - 6 A = 0$$

- Now you have three equations in three unknowns:

$$\begin{aligned} I_2 + I_3 &= I_1 \\ +6 A - 4I_2 - 2I_1 &= 0 \\ +12 A - 6I_3 + 4I_2 - 6 A &= 0 \end{aligned}$$

- If you substitute the top equation for I_1 in the second equation, you get

$$\begin{aligned} +6 A - 2(I_2 + I_3) - 4I_2 &= 0 \\ +12 A - 6I_3 + 4I_2 - 6 A &= 0 \end{aligned}$$

So:

$$\begin{aligned} +6 A - 2 I_3 - 6I_2 &= 0 \\ +6 A + 4I_2 - 6I_3 &= 0 \end{aligned}$$

- You can find I_3 in terms of I_2 by using the first equation:

$$I_3 = 3 A - 3I_2$$

- Then you can substitute this value of I_3 in the second equation to get

$$+6 A + 4I_2 - 6(3 - 3I_2) = 0$$

So:

$$-12 \text{ A} + 22I_2 = 0$$

And:

$$I_2 = 12 \text{ A} / 22 = 6 \text{ A} / 11 = 0.55 \text{ amp}$$

8. You now have one of the currents: $I_2 = 6/11$ amp. Plug that into:

$$+6 \text{ A} - 4I_2 - 2I_1 = 0$$

Giving you:

$$+6 \text{ A} - 4(6/11 \text{ A}) - 2I_1 = 0$$

Or, dividing by 2:

$$+3 \text{ A} - I_1 - 12/11 \text{ A} = 0$$

So:

$$I_1 = 21/11 \text{ A} = 1.91 \text{ amps}$$

9. Now you can find I_3 . Start with:

$$I_2 + I_3 = I_1$$

Which means:

$$I_3 = I_1 - I_2$$

So:

$$I_3 = \frac{21}{11} \text{ A} - \frac{6}{11} \text{ A} = \frac{15}{11} \text{ A} = 1.36 \text{ amp}$$

17 If R_1 is 3Ω , R_2 is 6Ω , and R_3 is 9Ω , what are the three currents in Figure 16-4, in which the batteries are 6.0 V and 12.0 V ?

18 If R_1 is 5Ω , R_2 is 10Ω , and R_3 is 15Ω , what are the three currents in Figure 16-4, in which the batteries are 6.0 V and 12.0 V ?

Answers to Problems about Circuits

The following are the answers to the practice questions presented earlier in this chapter. You'll see the answers worked out, step by step.

1 3.75×10^{19} electrons

1. Use this equation:

$$I = \frac{q}{t}$$

2. Solve for q :

$$q = It$$

3. Find the total charge passing in 1 second:

$$q = (6.0 \text{ A})(1.0 \text{ s}) = 6.0 \text{ C}$$

4. Divide by the charge of one electron to find the total number:

$$\text{Number} = \frac{6.0 \text{ C}}{1.6 \times 10^{-19}} = 3.75 \times 10^{19}$$

2 720 coulombs

1. Use this equation:

$$I = \frac{q}{t}$$

2. Solve for q :

$$q = It$$

3. Find the total charge passing in 1 minute:

$$q = (12.0 \text{ A})(60 \text{ s}) = 720 \text{ C}$$

3 4.0 amps

1. Use this equation:

$$V = IR$$

2. Solve for I :

$$I = \frac{V}{R}$$

3. Find the current:

$$I = \frac{V}{R} = \frac{6.0 \text{ V}}{1.5 \Omega} = 4.0 \text{ A}$$

4 12.0 amps

1. Use this equation:

$$V = IR$$

2. Solve for I :

$$I = \frac{V}{R}$$

3. Find the current:

$$I = \frac{V}{R} = \frac{6.0 \text{ V}}{0.5 \Omega} = 12.0 \text{ A}$$

5 **3.0 V**

1. Use this equation:

$$V = IR$$

2. Plug in the numbers:

$$V = I \cdot R = (1.0 \text{ A})(3.0 \Omega) = 3.0 \text{ V}$$

6 **7.5 V**

1. Use this equation:

$$V = IR$$

2. Plug in the numbers:

$$V = I \cdot R = (1.5 \text{ A})(5.0 \Omega) = 7.5 \text{ V}$$

7 **2,000 watts**

1. Use this equation:

$$P = I^2 R$$

2. Plug in the numbers:

$$P = I^2 R = (2.0 \text{ A})^2 (500 \Omega) = 2,000 \text{ watts}$$

8 **4,500 watts**

1. Use this equation:

$$P = I^2 R$$

2. Plug in the numbers:

$$P = I^2 \cdot R = (1.5 \text{ A})^2 (2,000 \Omega) = 4,500 \text{ watts}$$

9 **0.0375 A**

1. Use this equation:

$$R = R_1 + R_2$$

2. Plug in the numbers:

$$R = R_1 + R_2 = 60 \Omega + 100 \Omega = 160 \Omega$$

3. Use this equation:

$$V = IR$$

4. Solve for I :

$$I = \frac{V}{R}$$

5. Plug in the numbers:

$$I = \frac{V}{R} = \frac{6.0 \text{ V}}{160 \ \Omega} = 0.0375 \text{ A}$$

10 **0.020 A**

1. Use this equation:

$$R = R_1 + R_2$$

2. Plug in the numbers:

$$R = R_1 + R_2 = 100 \ \Omega + 200 \ \Omega = 300 \ \Omega$$

3. Use this equation:

$$V = IR$$

4. Solve for I :

$$I = \frac{V}{R}$$

5. Plug in the numbers:

$$I = \frac{V}{R} = \frac{6.0 \text{ V}}{300 \ \Omega} = 0.02 \text{ A}$$

11 **2.0 V**

1. Use this equation:

$$R = R_1 + R_2$$

2. Plug in the numbers:

$$R = R_1 + R_2 = 100 \ \Omega + 200 \ \Omega = 300 \ \Omega$$

3. Use this equation:

$$V = IR$$

4. Solve for I :

$$I = \frac{V}{R}$$

5. Plug in the numbers:

$$I = \frac{V}{R} = \frac{6.0 \text{ V}}{300 \ \Omega} = 0.02 \text{ A}$$

6. Use this equation:

$$V_1 = IR_1$$

7. Plug in the numbers:

$$V_1 = IR_1 = (0.02 \text{ A})(100 \Omega) = 2.0 \text{ V}$$

12 **4.0 V**

1. Use this equation:

$$R = R_1 + R_2$$

2. Plug in the numbers:

$$R = R_1 + R_2 = 100 \Omega + 200 \Omega = 300 \Omega$$

3. Use this equation:

$$V = IR$$

4. Solve for I :

$$I = \frac{V}{R}$$

5. Plug in the numbers:

$$I = \frac{V}{R} = \frac{6.0 \text{ V}}{300 \Omega} = 0.02 \text{ A}$$

6. Use this equation:

$$V_2 = IR_2$$

7. Plug in the numbers:

$$V_2 = IR_2 = (0.02 \text{ A})(200 \Omega) = 4.0 \text{ V}$$

13 **0.26 A**

1. Use this equation:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

2. Plug in the numbers:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{30 \Omega} + \frac{1}{90 \Omega} = 0.044 / \Omega$$

3. Find the total resistance:

$$R = \frac{1}{0.044 / \Omega} = 22.5 \Omega$$

4. Use this equation:

$$V = IR$$

5. Solve for I :

$$I = \frac{V}{R}$$

6. Plug in the numbers:

$$I = \frac{V}{R} = \frac{6.0 \text{ V}}{22.5 \Omega} = 0.26 \text{ A}$$

14 **0.18 A**

1. Use this equation:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

2. Plug in the numbers:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{45 \Omega} + \frac{1}{120 \Omega} = 0.03 / \Omega$$

3. Find the total resistance:

$$R = \frac{1}{0.03 / \Omega} = 33 \Omega$$

4. Use this equation:

$$V = IR$$

5. Solve for I :

$$I = \frac{V}{R}$$

6. Plug in the numbers:

$$I = \frac{V}{R} = \frac{6.0 \text{ V}}{33 \Omega} = 0.18 \text{ A}$$

15 **0.22 A**

1. Use this equation:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

2. Plug in the numbers:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{39 \Omega} + \frac{1}{93 \Omega} = 0.036 / \Omega$$

3. Find the total resistance:

$$R = \frac{1}{0.036 / \Omega} = 27 \Omega$$

4. Use this equation:

$$V = IR$$

5. Solve for I :

$$I = \frac{V}{R}$$

6. Plug in the numbers:

$$I = \frac{V}{R} = \frac{6.0 \text{ V}}{27 \Omega} = 0.22 \text{ A}$$

16 **0.25 A**

1. Use this equation:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

2. Plug in the numbers:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{42 \Omega} + \frac{1}{56 \Omega} = 0.042 / \Omega$$

3. Find the total resistance:

$$R = \frac{1}{0.042 / \Omega} = 24 \Omega$$

4. Use this equation:

$$V = IR$$

5. Solve for I :

$$I = \frac{V}{R}$$

6. Plug in the numbers:

$$I = \frac{V}{R} = \frac{6.0 \text{ V}}{24 \Omega} = 0.25 \text{ A}$$

17 $I_1 = 42/33 = 1.27 \text{ amps}$, $I_2 = 12/33 = 0.36 \text{ amp}$, $I_3 = 30/33 = 0.91 \text{ amp}$

1. $\Sigma I_{in} = \Sigma I_{out}$ at any point, so use point A at left in Figure 16-4. I_2 and I_3 flow into it, and I_1 flows out of it, so:

$$I_2 + I_3 = I_1$$

2. $\Sigma V = 0$ around a loop. Go around the two internal loops. From the top loop, you get

$$+6 \text{ V} - (3 \Omega)I_1 - (6 \Omega)I_2 = 0$$

3. From the bottom loop, you get

$$+12 \text{ V} - 6 \text{ V} + (6 \Omega)I_2 - (9 \Omega)I_3 = 0$$

4. You have three equations in three unknowns (after you divide the second and third equation by 1Ω):

$$\begin{aligned} I_2 + I_3 &= I_1 \\ 6 \text{ A} - 3I_1 - 6I_2 &= 0 \\ 6 \text{ A} + 6I_2 - 9I_3 &= 0 \end{aligned}$$

5. If you substitute the top equation for I_1 in the second equation, you get

$$6 \text{ A} - 3(I_2 + I_3) - 6I_2 = 0$$

$$6 \text{ A} + 6I_2 - 9I_3 = 0$$

So:

$$6 \text{ A} - 3I_3 - 9I_2 = 0$$

$$6 \text{ A} + 6I_2 - 9I_3 = 0$$

6. You can find I_3 in terms of I_2 by using the first equation:

$$I_3 = 2 \text{ A} - 3I_2$$

7. Then you can substitute this value of I_3 in the second equation, giving you:

$$6 \text{ A} + 6I_2 - 9(2 \text{ A} - 3I_2) = 0$$

So:

$$-12 \text{ A} + 33I_2 = 0$$

And:

$$I_2 = 12/33 \text{ A} = 0.36 \text{ amp}$$

8. Plug I_2 into:

$$+6 \text{ A} - 3I_1 - 6I_2 = 0$$

Giving you:

$$+6 \text{ A} - 3I_1 - 6(12/33) = 0$$

Or, dividing by 3:

$$+2 \text{ A} - I_1 - 24/33 \text{ A} = 0$$

So:

$$I_1 = 42/33 \text{ A} = 1.27 \text{ amps}$$

9. Now you can find I_3 . Start with:

$$I_2 + I_3 = I_1$$

Which means:

$$I_3 = I_1 - I_2$$

So:

$$I_3 = \frac{42}{33} \text{ A} - \frac{12}{33} \text{ A} = \frac{30}{33} \text{ A} = 0.91 \text{ amp}$$

18 $I_1 = 42/55 = 0.76 \text{ amp}$, $I_2 = 12/55 = 0.22 \text{ amp}$, $I_3 = 30/55 = 0.55 \text{ amp}$

1. $\Sigma I_{in} = \Sigma I_{out}$ at any point, so use point A at left in Figure 16-4. I_2 and I_3 flow into it, and I_1 flows out of it, so:

$$I_2 + I_3 = I_1$$

2. $\Sigma V = 0$ around a loop. Go around the two internal loops. From the top loop, you get

$$+6 \text{ V} - (5 \Omega)I_1 - (10 \Omega)I_2 = 0$$

3. From the bottom loop, you get

$$+12 \text{ V} - 6 \text{ V} + (10 \Omega)I_2 - (15 \Omega)I_3 = 0$$

4. You have three equations in three unknowns (after you divide the second and third equation by 1 Ω):

$$\begin{aligned} I_2 + I_3 &= I_1 \\ +6 \text{ A} - 5I_1 - 10I_2 &= 0 \\ 6 \text{ A} + 10I_2 - 15I_3 &= 0 \end{aligned}$$

5. If you substitute the top equation for I_1 in the second equation, you get

$$\begin{aligned} 6 \text{ A} - 5(I_2 + I_3) - 10I_2 &= 0 \\ 6 \text{ A} + 10I_2 - 15I_3 &= 0 \end{aligned}$$

So:

$$\begin{aligned} +6 \text{ A} - 5I_3 - 15I_2 &= 0 \\ 6 \text{ A} + 10I_2 - 15I_3 &= 0 \end{aligned}$$

6. You can find I_3 in terms of I_2 by using the first equation:

$$I_3 = \frac{6}{5} \text{ A} - 3I_2$$

7. Then you can substitute this value of I_3 in the second equation, giving you:

$$6 \text{ A} + 10I_2 - 15\left(\frac{6}{5} \text{ A} - 3I_2\right) = 0$$

So:

$$-12 \text{ A} + 55I_2 = 0$$

And:

$$I_2 = 12/55 \text{ A} = 0.22 \text{ amp}$$

8. Plug I_2 into:

$$+6 \text{ A} - 5I_1 - 10I_2 = 0$$

Giving you:

$$+6 \text{ A} - 5I_1 - 10(12/55) \text{ A} = 0$$

Or, dividing by 5:

$$+\frac{6}{5} \text{ A} - I_1 - \frac{24}{55} \text{ A} = 0$$

So:

$$I_1 = 42/55 \text{ A} = 0.76 \text{ amp}$$

9. Now you can find I_3 . Start with:

$$I_2 + I_3 = I_1$$

Which means:

$$I_3 = I_1 - I_2$$

So:

$$I_3 = \frac{42}{55} \text{ A} - \frac{12}{55} \text{ A} = \frac{30}{55} \text{ A} = 0.55 \text{ amp}$$

6

The Part of Tens

IN THIS PART . . .

Find out what the ten most-common physics problem-solving mistakes are.

Check out ten wild physics theories.

- » Handling common mistakes
- » Knowing how to spot potential problems

Chapter **17**

Ten Common Mistakes People Make When Solving Problems

This chapter discusses the most common errors people make when working out physics problems. Be sure to pay attention to certain types of problems that will stand out as noted here.

Mixing Units

The most common error made in solving physics problems involves mixing the units from one system with another system. If the problem is given to you in inches, kilograms, and seconds, convert it into a consistent system of units before proceeding to work out the answer. For example, if you want to use the MKS system, convert *everything* into MKS before working out the problem.

Expressing the Answer in the Wrong Units

If the problem asks for the answer in the MKS system, don't give it in CGS units. You'd be surprised at how common an error this is; people are so relieved that they've solved the problem that they goof it up in the last step.

Swapping Radians and Degrees

Degrees are commonly used in physics problems — except when it comes to angular velocity and acceleration. That's when you have to make sure you're working with radians. Use the $180^\circ/\pi$ conversion factor to convert from radians to degrees when needed.

Getting Sines and Cosines Mixed Up

Physics students often make the mistake of interchanging sines and cosines. Take a look at Figure 17-1 and keep the following relationships in mind:

$$\sin \theta = y/h = \text{opposite/hypotenuse}$$

$$\cos \theta = x/h = \text{adjacent/hypotenuse}$$

$$\tan \theta = y/x = \text{opposite/adjacent}$$

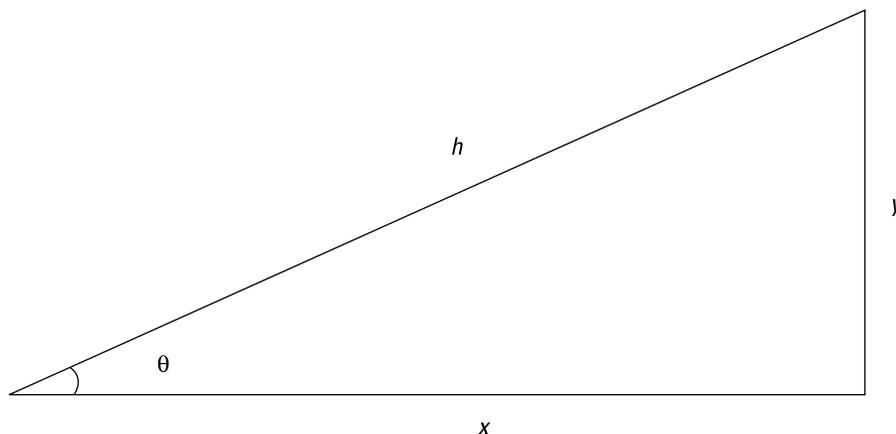


FIGURE 17-1:
A triangle.

© John Wiley & Sons, Inc.

Failing to Treat Vectors as Vectors

When you add vectors, use vector addition. That means you are resolving vectors into components. Many people will just add the magnitudes of the vectors without realizing that they should be adding components instead.

Neglecting Latent Heat

When you're faced with a problem that involves a phase change, such as from ice to water, don't forget to take the latent heat into account. When ice changes into water, it absorbs latent heat that you have to account for in your solution.

Getting the Direction of Forces Wrong

When you are solving a force problem, you can keep track of the directions of the forces by drawing a free-body diagram. The gravitational force exerted by the Earth always points towards the center of the Earth. The normal force is always perpendicular to the surface (not necessarily up). The frictional force opposes the relative motion of surfaces in contact.

Getting the Signs Wrong in Kirchhoff Loops

You use Kirchhoff's laws to solve for the currents in a circuit, but many people run into trouble with Kirchhoff's laws problems because they get the signs wrong.



TIP

To be sure you get the signs right, put in arrows for all the currents. Don't worry about getting the direction wrong for an arrow; if you do, the current will just come out negative. Then put a plus sign (+) where the current enters each resistor and a negative sign (-) where the current leaves each resistor, as shown in Figure 17-2.

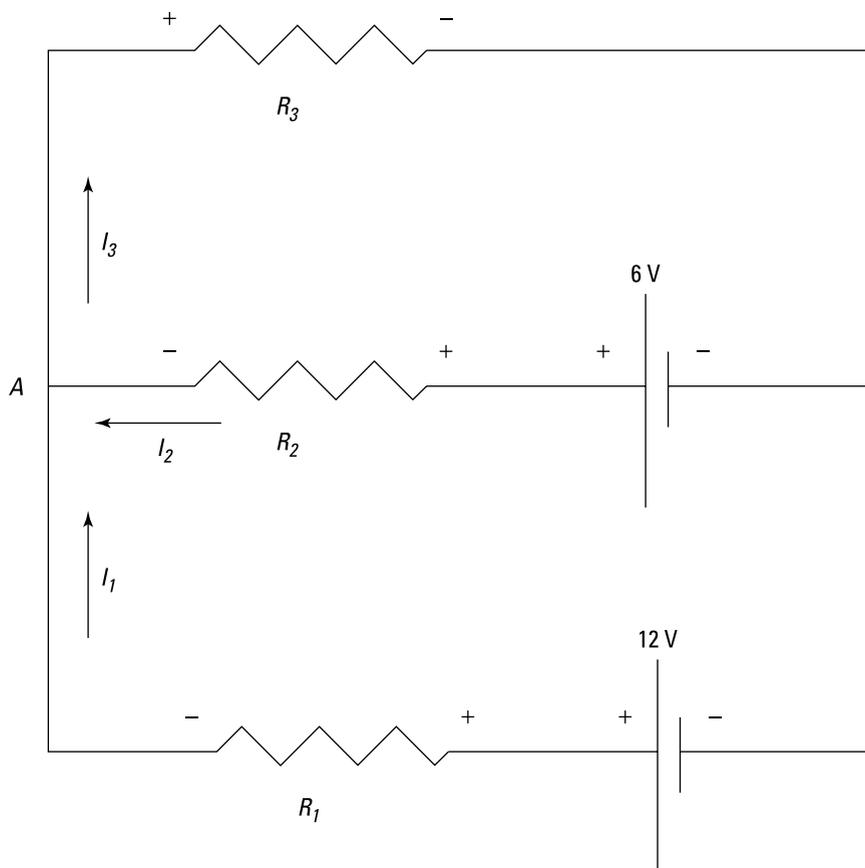


FIGURE 17-2: A circuit.

© John Wiley & Sons, Inc.

Adding Resistors Incorrectly

When you have resistors in series, the current has to pass through one after the other. Here's how you calculate the total resistance of two resistors in a series:

$$R = R_1 + R_2$$

When you have two resistors in parallel, the current divides between the two of them, and you add the resistors, like this:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Many people get these two confused — make sure you don't.

Using the Wrong Temperature in the Ideal Gas Law

When you use the ideal gas law, always convert the temperature to Kelvins. If you are given the temperature in Celsius, just add 273.15 to get a temperature in Kelvins.

IN THIS CHAPTER

- » Pulling apart a proton
- » Mixing mass and energy
- » Coming to grips with changing constants
- » Tunneling through things

Chapter **18**

Ten Wild Physics Theories

This chapter gives you ten crazy-but-true physics facts. Very small, dense, or fast objects behave very differently from the objects you encounter every day. The physics theories you use to describe everyday objects are approximations to the more complete theories that explain the results of experiments involving smaller, denser, faster, or otherwise exotic objects. Don't be shocked if one day some of the theories described here turn out to be approximations to a better theory developed in the future.

The Universal Speed Limit

The fastest thing in the universe is light traveling through empty space, or vacuum. Its speed is exactly 299,792,458 meters per second, or about 186,000 miles per second. According to Albert Einstein's theory of relativity, you get more massive as you move faster. At everyday speeds, your mass doesn't change much. Your mass doubles once you reach about 87% of the speed of light and keeps increasing the faster you go. As you approach the speed of light, it gets harder and harder to accelerate due to your increased mass. The universe has no need for speeding tickets!

Through the Looking Glass, and What Chien-Shiung Found There

Can you tell the difference between the universe you live in, and a mirror universe where left and right are switched? It isn't as simple as looking for living chess pieces or Tweedledum and Tweedledee. Most of the laws of physics work the same in the mirror universe. If you set up an experiment and watch it in a mirror, the results will be the same as the results of a mirrored copy of the experiment — unless your experiment involves the weak nuclear force, which is responsible for the radioactive decay of atoms. Chien-Shiung Wu showed us in 1956 that when the weak nuclear force is involved, a mirrored copy does not behave like a mirror image. What a strange world we live in!

Wanted: Dead and Alive

You may have heard of Schrödinger's cat. Erwin Schrödinger proposed a thought experiment to illustrate quantum superposition, or the ability of a quantum system to be in two (or more) states simultaneously until it's measured. Schrödinger imagined placing a cat in a sealed box with some poison and a radioactive source. The radioactive source has a chance of decaying. If the source decays, the poison is released, and the cat dies. Otherwise, the cat lives. Until the box is opened, the cat is in a superposition — it is both dead and alive!

So far scientists have only figured out how to create a quantum superposition with smaller things like electrons, atoms, or molecules. Perhaps someday someone will figure out how to superpose something as large as a cat.

Quantum Objects Can Tunnel

If you throw a ball against a wall, it will bounce off. Well, at least most of the time it will bounce off. There is a very small probability that the ball will *tunnel* through the wall. One moment the ball will be on one side of the wall, and the next the ball will be on the other side of the intact wall. The smaller and faster the ball (or particle) is, the larger the probability of tunneling. Electrons, which are quite small, tunnel through barriers in a variety of electronic devices including some computers.

Mass Is a Kind of Energy

The most famous physics equation of all is Albert Einstein's

$$E = mc^2$$

Or energy equals mass times the speed of light squared. Einstein's equation tells you that mass is just another type of energy. So other types of energy, like kinetic energy or potential energy, can be converted into mass. If you smash together two particles that have a lot of kinetic energy, you may end up with your original two particles plus some additional massive particles.

It can also work in reverse — you can convert mass into energy. Nuclear power plants produce energy this way. It doesn't take much mass to make a huge amount of energy — six grams can power New York City for a day!

Vacuum Is Not Just Empty Space

The vacuum is the lowest energy state. That vacuum has no particles. That is, it has no real particles that can be measured directly by a particle detector. It's not quite just empty space though. Virtual particles are constantly popping up and disappearing. These virtual particles don't stick around long enough to be measured by particle detectors. If you send a real particle through the vacuum, the motion of the real particle can be affected by the virtual particles.

"Constants" in Physics Change

Most of the constants in physics aren't really constant; they depend on the energy of the probe you use to measure them.

For example, the charge of an electron is $-e = -1.602 \times 10^{-19}$ C if you measure it at low energy. At the highest energies you can reach using current technology, the magnitude of the charge you measure increases by a few percent. Every electron is surrounded by a cloud of virtual charged particles. The positively charged virtual particles like to hang out closer to the electrons than the negatively charged virtual particles. If you measure the charge of the electron from far away, you're also measuring some of the positively charged cloud of virtual particles near the electron. So the charge of the electron is screened or hidden by the virtual particles. Higher energy probes get closer to the electron. The closer to the electron the probe is, the less of the cloud you measure and the larger the magnitude of the charge of the electron you will see.

Stuck in the Middle of a Proton

Protons are *hadrons*, which means they are made up of smaller particles called *quarks* and *gluons*. The quarks and gluons inside a hadron are stuck together so well that no one has ever seen an isolated quark or gluon. Why not?

Hadrons are held together by the strong nuclear force. Unlike the electric or gravitational forces that get weaker at longer distances, the strong nuclear force between two quarks gets stronger the farther apart they are. If you could grab onto one quark in each hand and pull, one of two things would happen. You might get tired of pulling. When you let go, the quarks would happily snuggle together inside a single hadron. Or, if you're strong enough, you could

pull so hard that there's enough energy to convert into a quark-antiquark pair. The new quark pairs up with the quark in one hand, and the new antiquark gets together with the quark in the other hand. In the end, you'd be holding a hadron in each hand!

The Expansion of the Universe Is Accelerating

The universe is expanding. That means that the distance between you and distant galaxies tends to increase. Things that are gravitationally bound, like you and the Earth, or the Earth and the sun, are nearly unaffected by the expansion of the universe.

Distant galaxies are receding from you faster and faster every day, which tells you the expansion of the universe is accelerating. Light moves very quickly, but it takes some time to travel from there to here. So the farther away you look, the earlier the light was emitted. You can look back in time by looking far away!

Some supernova, or exploding stars, are known as standard candles because they always emit the same amount of light. Far away candles or supernovae appear dimmer. By measuring the brightness of these supernova, you can determine how far away they are and how long ago the stars exploded. The color of the supernova tells you about how fast the star was moving. By looking at lots of supernova, you can determine how fast the universe was expanding over the last few billion years. You'll find that the expansion of the universe is speeding up.

Some Things Never Change

If you measure the mass of the electron in your backyard laboratory tomorrow, you'll get the same result as a Martian settler will measure 400 years from now. This fact is known as translation symmetry — physics experiments give the same results no matter where or when they are performed.

In 1915, mathematician Emmy Noether proved that symmetries of the laws of physics, or ways you can change a physics experiment without affecting its results, lead to conservation laws. Noether's Theorem tells you that translation symmetry implies that energy and momentum must be conserved. If you identify the symmetries of some physical system, you know a lot about how that system evolves in time.

Index

Symbols

- α (angular acceleration), 106–110
- ω (angular velocity), 171
- β (coefficient of volume expansion), 230
- a (linear acceleration), 107
- Ω (ohms), 307
- ϵ (permittivity of free space), 286
- δ (Stefan-Boltzman constant), 248
- τ (torque), 173

A

- absolute zero, 226
- acceleration
 - about, 29–30
 - angular, 108–110, 172–173
 - calculating, 67–69, 210–211
 - centripetal, 110–112
 - displacement, 31–34
 - inclined planes, 84–85, 93–94
 - simple harmonic motion, 210–211
 - time, 31–33
 - velocity, 33–34
- adiabatic processes, 268–270
- algebra, 18–19
- Ampere (A), 8, 305
- amplitude, 205
- angles
 - angular velocity, 104–106, 108–110, 171–172
 - converting between, 101–103
 - angular acceleration (α)
 - about, 108–110
 - angles, connecting to, 108–110
 - angular velocity, connecting to, 108–110
 - unit of measure, 106–107
 - as a vector, 172–173
 - angular frequency, harmonic motion, 207–213
 - angular momentum (L), 195–196

- angular speed, 205
- angular velocity
 - about, 104–106
 - angles, connecting to, 108–110
 - angular acceleration, 108–110
 - as a vector, 171–172
- Avogadro's Number (N), 250–251

B

- barye (Ba), 8
- biot (Bi), 8
- Boltzmann's constant (k), 251–254
- Btu (British Thermal Unit), 12, 232

C

- C (Coulomb), 8, 11, 285, 289, 293, 305
- Calorie (C), 232
- calories compared with Calories, 232
- Carnot heat engines, 274–275
- Celsius (C) scale, 230
- centigrade scale. *See* Celsius (C) scale
- centimeter (cm), 8
- centripetal acceleration, 110–112
- centripetal force, 112–113
- CGS (centimeter-gram-second) system
 - conversions, 12
 - force, measuring, 62
 - torque, measuring, 173–176
 - units of measure, 8, 62
 - work, measuring, 127–131
- charge density, 291
- Cheat Sheet (website), 2
- Chien-Shiung Wu, 334
- circuits. *See* electric circuits
- circular motion. *See also* frequency; period (T)
 - about, 101

- angular acceleration, 108–110
- angular velocity, 104–106
- centripetal force, 112–113
- frequency, 103–104
- period, 103–104
- circular motion, angles
 - angular acceleration, 108–110
 - angular velocity, 108–110
 - centripetal acceleration, 110–112
 - conversions, 101–103
- cm (centimeter), 8
- coefficient of friction
 - about, 87
 - kinetic, 89, 91–92
 - static, 90–91
- coefficient of linear expansion, 228
- coefficient of static friction, 88
- coefficient of volume expansion (β), 230
- collisions. *See also* kinetic energy
 - elastic, 251
 - two-dimensional, 156–158
- conduction, 245–247
- constants, 335
- convection, 244
- conversions
 - between angles, 101–103
 - BTUs to Joules, 232
 - CGS (centimeter-gram-second) system, 11–12
 - degrees/radians, 101–103
 - distance units, 11–12
 - FPI (foot-pound-inch) system, 11–12
 - measurement systems, 11–12
 - MKS (meter-kilogram-second) system, 11–12
 - between temperature scales, 225–227
 - through multiple units, 14–15
 - time units, 15–16
 - between units, 11–12

coordinate system for vectors, 43–44
cosine/sine mixups, 330
Coulomb (C), 8, 11, 285, 289, 293, 305
current (I), 305–306

D

deceleration, 30
degree/radian conversion, 101–103, 330
diameter (d), 103
direction. *See* vectors, direction
direction of force, 331
displacement
 about, 25–27
 acceleration, 31–34
 time, 31–33
 velocity, 33–34
distance. *See* displacement
distance units, conversions, 11–12
drag, inclined planes. *See* inclined planes, friction
dyne, 8

E

Einstein, Albert, 334–335
elastic collisions, 251
electric charge
 Coulomb (C), 8, 11, 285, 289, 293, 305
 negative, 285
 positive, 285
 unit of measure, 11
electric circuits
 Ampere (A), 305
 current, 305–306
 current (I), 305–306
 junction rule, 314
 Kirchoff's Rules, 314–316, 331
 loop rule, 314
 ohms (Ω), 307
 Ohm's Law, 306–308
 parallel, 311–313
 power, 12, 308–309
 resistance (R), 306–308

 series, 309–311
electric circuits, resistors
 common mistakes, 332
 in parallel, 311–313
 in series, 309–311
electric fields. *See also* electric circuits
 about, 289–290
 charge density, 291
 parallel plate capacitors, 291–292
 point charges, 291, 294
 voltage, 293–294
electric forces
 about, 286–287
 permittivity of free space, 286
 as vectors, 288–289
electric potential. *See* voltage
electricity, static
 electric charges, 285–286
 electric fields, 289–292
 electric potential, 294–296
 electrical force, 286–289
 parallel plate capacitors, 291–292
 voltage, 293–294
electromotive force (EMF), 305
emissivity, 248
energy
 calculating, 12
 kinetic, 114, 127–131, 154–155, 191–192
 mass as a type of, 334–335
 potential, 132–133
 simple harmonic motion, 213–214
equilibrium, 71–73, 176–179
erg, 8, 11, 12, 114
errors, in problem solving, 329–332
Example icon, 2

F

Fahrenheit (F) scale, 225
First Law of Thermodynamics, 261–262
foot-pound, 174. *See also* FPI (foot-pound-inch) system
force. *See also* Newton's Three Laws of Motion; torque (τ)
 acceleration, calculating, 67–69

 centripetal, 112–113
 direction of, 331
 electrical, 286–289
 equilibrium, 71–73
 of gravity, 69, 71–73
 inertia, 61
 mass, unit of measure, 69
 mass compared with weight, 69, 71–73
 net, calculating, 67–69, 125–127
 normal, 82
 units of measurement, 12, 62
 as a vector, 65–67
FPI (foot-pound-inch) system, 7, 11–12, 174
FPS (foot-pound-second) system
 force, measuring, 62
 slugs, 62
franklin (Fr), 8
frequency, 103–104, 207–208
friction
 inclined planes. *See* inclined planes, friction
 kinetic, 89, 91–92
 static, 90–91

G

gases
 Boltzmann's constant (k), 251–254
 elastic collisions, 251
 ideal, 251–253
 universal gas constant (R), 251
gases, thermodynamic processes
 adiabatic, 268–270
 constant heat, 268–270
 constant pressure, 262–264
 constant temperature, 266–267
 constant volume, 265–266
 isobaric, 262–264
 isochoric, 265–266
 isothermal, 266–267
 isotherms, 268
gauge pressure, 252
gauss (G), 8
glucons, 335–336
gram (g), 8

gravity (g)
force of, on inclined planes,
82, 90, 94
mass compared with weight,
69, 71–73

H

hadrons, 335
harmonic motion (springs)
about, 203, 205–206
acceleration, 210–211
angular frequency, 207–213
angular speed, 205
energy, 213–214
frequency, 207–208
Hooke's Law, 203–204
pendulums, 214–215
periods, 207–208
potential energy, 213–214
springs, 211–213
velocity, 208–209
heat. *See* thermodynamics
heat capacity, 231–233
heat engines, 271–274
Hertz (Hz), 207
Hooke's Law, 203–204
hypotenuse, 47

I

I (current), 305–306
icons, explained, 2
ideal gas law, 251–253, 332
impulse. *See* kinetic energy, impulse
inclined planes
acceleration, 84–85, 93–94
breaking into vectors, 81–83
normal force, 82
rotational dynamics, 193–195
speed, 85–86
work, measuring, 129–130
inclined planes, friction
acceleration, 84–85, 93–94
coefficient of, 87
friction on, 87–89, 90–91, 91–92
kinetic, 91–92
kinetic, coefficient of, 89, 91–92

static, 90–91
static, coefficient of, 90–91
inclined planes, sliding objects
acceleration, 84–85, 93–94
coefficient of friction, 87
kinetic coefficient of friction, 89,
91–92
kinetic friction, 89, 91–92
speed of, 85–86
static coefficient of friction,
90–91
static friction, 90–91
inertia, 61
isobaric work, 262–264
isochoric processes, 265–266
isothermal processes, 266–267
isotherms, 268

J

Joules (J)
converting to Btus, 232
measuring work, 124–125
unit of measure, 8, 11
junction rule, 314

K

k (Boltzmann's constant), 252
Kelvin (K) scale, 226–227
kilogram (kg), 8, 11, 62
kilogram-meters² (kg-m²), 186
kinetic coefficient of friction, 89,
91–92
kinetic energy. *See also* potential
energy; work
about, 114, 127–129, 154–155
conserving, 154–155
elastic collisions, 251
getting from work, 129–131
rotational dynamics, 191–192
two-dimensional collisions,
156–158
kinetic energy, impulse
about, 147–148, 150–151
momentum, 150–153
unit of measure, 150
kinetic energy, momentum,
conserving, 152–153

impulse, 150–151
unit of measure, 152
kinetic friction, 89, 91–92
Kirchoff's Rules, 314–316, 331

L

L (angular momentum), 195–196
latent heat, 234–236, 330
Laws of Motion. *See* Newton's
Three Laws of Motion
length, units of measure, 12
light years, 26
linear acceleration (a), 107
linear expansion, 227–229
liquid-to-gas change, 234
loop rule, 314

M

magnetism, units of measure, 8
magnitude, finding of vectors,
46–48
mass
about, 62
compared with weight, 69–71
conversions, 11–12
as a type of energy, 334–335
units of measure, 7–9
measurement units. *See* units of
measure
meter (m), 8
MKS (meter-kilogram-second)
system
conversions, 11–12
force, measuring, 62
torque, measuring, 173–176
units of measure, 8, 62
work, measuring, 127–131
moment of inertia, 185, 187–189
momentum (p). *See also* kinetic
energy, momentum
about, 149–150
angular momentum (L), 195–196
conservation of, 152–153
relationship with impulse,
150–151
motion as vector, 51

N

N (Avogadro's Number), 250–251
negative electric charge, 285
net force, calculating, 67–69,
125–127. *See also* force
Newton (N), 8
Newton's Three Laws of Motion.
See also force
 First Law, 61–62
 Second Law, 62–65
 Third Law, 73
Newton-second, 148
Noether, Emmy, 336
Noether's Theorem, 336
normal force, 82
numbers
 rounding, 17
 scientific notation, 10–11
 significant figures, 16–17

O

ohms (Ω), 307
Ohm's Law, 306–308
origins of vectors, 45

P

p (momentum). *See* momentum (p)
parallel circuits, 311–313
parallel plate capacitors, 291–292
Pascal (P), 8
pendulums, 214–215
period (T), 103–104, 207–208
permittivity of free space (ϵ), 286
phase changes, 234
physical work. *See* work
physics. *See also specific topics*
 algebra, 18–19
 basics, 7–24
 measurements, 7–9
 multiple conversions, 14–15
 scientific notation, 10–11
 significant figures, 16–17
 time conversions, 15–16
 trigonometry, 19–20
 unit conversions, 11–12
point charges, 291, 294

positive electric charge, 285
potential energy, 132–133,
213–214. *See also* kinetic
energy
power, 12, 308–309
pressure, units of measure, 8
protons, 335–336
Pythagorean theorem, 47

Q

quantum objects, 334
quantum superposition, 334
quarks, 335–336

R

R (universal gas constant), 251
radian/degree conversion,
101–103, 330
radians, 101, 330
radiation, 248–249
radioactive decay, 334
radius (r), 103
ramps. *See* inclined planes
Remember icon, 2
resistance (R), 306–308
resistors, 309–313, 332
right triangles, 19–20
rotational dynamics. *See also*
torque (τ)
 about, 185–187
 angular momentum, 195–196
 inclined planes, 193–195
 kinetic energy, 191–192
 moment of inertia, 187–189
 work, 190–191
rotational equilibrium, 176–179
rotational kinematics
 angular acceleration, as a vector,
172–173
 angular velocity, as a vector,
171–172
 equilibrium, 176–179
 tangential acceleration, 170–171
 tangential speed, 167–169
 torque, 173–176
rotational kinetic energy, 191–192
rounding numbers, 17

S

scalar value, 149
scalars, 51
Schrödinger, Erwin, 334
scientific notation, 10–11
second (s), 8
Second Law of
Thermodynamics, 271
series circuits, 309–311
significant figures, 16–17
sine/cosine mixups, 330
slug, 62
solid-to-gas change, 234
solid-to-liquid change, 234
specific heat capacity, 232
speed. *See also* acceleration;
velocity
 about, 27
 angular, 205
 on inclined planes, 85–86
 relating to acceleration, 33–34
 tangential, 167–169
speed theory, 333
spring constant, 203, 211–213
spring motion. *See* harmonic
motion (springs)
static coefficient of friction, 90–91
static electricity
 about, 285
 electric charges, 285–286
 electric fields, 289–292
 electric potential, 294–296
 electrical force, 286–289
 parallel plate capacitors, 291–292
 voltage, 293–294
static friction, 90–91
Stefan-Boltzman constant (δ), 248
storing energy. *See* potential energy

T

T (period), 103–104, 207–208
tangential acceleration, 170–171
tangential speed, 167–169
Technical Stuff icon, 2
temperature scales, converting,
225–227
Tesla (T), 8

theories, 333–336
 thermal energy, 243
 thermal expansion, 227
 thermodynamics. *See also* gases
 absolute zero, 226
 Avogadro's Number, 250–251
 Btu (British Thermal Unit), 12, 232
 calories compared with
 Calories, 232
 Celsius (C) scale, 230
 coefficient of volume
 expansion, 230
 constant, 268–270
 converting between temperature
 scales, 225–227
 defined, 225
 first law, 261–262
 heat capacity, 231–233
 ideal gas, 251–253
 Kelvin (K) scale, 226–227
 latent heat, 234–236, 330
 linear expansion, 227–229
 liquid-to-gas change, 234
 phase changes, 234
 second law, 271
 solid-to-gas change, 234
 solid-to-liquid change, 234
 specific heat capacity, 232
 thermal expansion, 227
 third law, 275
 volume expansion, 230–231
 thermodynamics, heat transfer
 Carnot heat engines, 274–275
 conduction, 245–247
 convection, 244
 direction of flow, 271
 emissivity, 248
 heat engines, 271–274
 radiation, 248–249
 Stefan-Boltzman constant (δ), 248
 Third Law of Thermodynamics, 275
 time
 connecting with acceleration and
 displacement, 31–33
 converting, 15–16
 relating to displacement, 31–33

unit conversion, 11
 unit of measure, 7–9
 Tip icon, 2
 torque (τ). *See also* force; rotational
 dynamics
 rotational kinematics, 173–176
 unit of measure, 173–176
 trigonometry, 19–20
 two-dimensional collisions, 156–158

U

units of measure. *See also* CGS
 (centimeter-gram-second)
 system; FPI (foot-pound-
 inch) system; FPS (foot-pound-
 second) system; MKS
 (meter-kilogram-second)
 system
 angular acceleration, 106–107
 common mistakes, 329
 expressing answers in
 incorrect, 329
 force, 12
 of force, 62
 frequency, 207
 Hertz (Hz), 207
 impulse, 150
 mixing, 329
 moment of inertia, 185,
 187–189
 momentum, 152
 Newton-second, 148
 torque, 173–176
 work, 127–131
 units of measure, conversions
 converting between, 11–15
 distance, 11–12
 multiple, 14–15
 between systems, 11–12
 time, 15–16
 universal gas constant (R), 251
 universe expansion, 336

V

vacuum, 335
 vectors

about, 41
 adding, 48–51
 angular acceleration as, 172–173
 angular velocity as, 171–172
 breaking inclined planes into,
 81–83
 common mistakes, 330
 components of, 43–46
 coordinate system for, 43–44
 force as, 65–67
 motion as, 51
 origins, 45
 vectors, direction
 about, 41
 converting to coordinate system,
 43–44
 creating a vector, 41–43
 finding, 46–48
 vectors, magnitude
 about, 41
 converting to coordinate system,
 43–44
 creating a vector, 41–43
 finding, 46–48
 velocity. *See also* speed
 about, 27, 208–209
 angular, 104–106, 108–110,
 171–172
 connecting with acceleration and
 displacement, 33–34
 simple harmonic motion, 208–209
 as a vector, 51–53
 voltage, 293–294
 volume expansion, 230–231,
 265–266

W

weight, 69–71
 work. *See also* kinetic energy
 about, 114
 measuring, 127–131
 net force, 125–127
 potential energy, 132–133
 power, 134–135
 work dynamics, rotational, 190–191

Publisher's Acknowledgments

Acquisitions Editor: Greg Tubach

Project Editor: Donna Wright

Technical Editors: Dan Wohns, Amy Nicklin

Managing Editor: Michelle Hacker

Cover Image: © Mopic /Fotolia

Production Editor: Tamilmani Varadharaj

WILEY END USER LICENSE AGREEMENT

Go to www.wiley.com/go/eula to access Wiley's ebook EULA.