coursera

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Vector Identities

Prove the following equalities:

(a) $\boldsymbol{\nabla} \times \{(\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u}\} = \boldsymbol{\nabla} \times (\boldsymbol{\omega} \times \boldsymbol{u})$;

(b) $\boldsymbol{\nabla} \times (\boldsymbol{\omega} \times \boldsymbol{u}) = (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{\omega} - (\boldsymbol{\omega} \cdot \boldsymbol{\nabla})\boldsymbol{u}$.

You can use the facts that the curl of a gradient and the divergence of a curl are equal to zero, and the general vector identity

$$\boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{v}) = \boldsymbol{u}(\boldsymbol{\nabla} \cdot \boldsymbol{v}) - \boldsymbol{v}(\boldsymbol{\nabla} \cdot \boldsymbol{u}) + (\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{u} - (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{v}.$$

You will also need to use $\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{u}$ and $\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$. However, you will need to prove that

$$\boldsymbol{u} \times (\boldsymbol{\nabla} \times \boldsymbol{u}) = \tfrac{1}{2}\boldsymbol{\nabla}(\boldsymbol{u} \cdot \boldsymbol{u}) - (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u}.$$

Note: Remember, you may check the solutions in the lecture notes.

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