1 Trigonometry

1.1 Classical identities

- $\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$.
- $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$.
- $\operatorname{tg}(x \pm y) = \frac{\operatorname{tg} x \pm \operatorname{tg} y}{1 \mp \operatorname{tg} x \operatorname{tg} y}$.
- $\operatorname{ctg}(x \pm y) = \frac{\operatorname{ctg} x \operatorname{ctg} y \mp 1}{\operatorname{ctg} y \pm \operatorname{ctg} x}$.
- $\sin 2x = 2 \sin x \cos x$.
- $\cos 2x = \cos^2 x - \sin^2 x$.
- $\operatorname{tg} 2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}$.
- $\operatorname{ctg} 2x = \frac{\operatorname{ctg}^2 x - 1}{2 \operatorname{ctg} x}$.
- $\sin x \sin y = \frac{\cos(x-y) - \cos(x+y)}{2}$.
- $\sin x \cos y = \frac{\sin(x-y) + \sin(x+y)}{2}$.
- $\cos x \cos y = \frac{\cos(x-y) + \cos(x+y)}{2}$.
- $\operatorname{tg} x \operatorname{tg} y = \frac{\cos(x-y) - \cos(x+y)}{\cos(x-y) + \cos(x+y)}$.
- $\operatorname{tg} x \operatorname{ctg} y = \frac{\sin(x-y) + \sin(x+y)}{\sin(x+y) - \sin(x-y)}$.
- $\operatorname{ctg} x \operatorname{ctg} y = \frac{\cos(x-y) + \cos(x+y)}{\cos(x-y) - \cos(x+y)}$.
- $\sin^2 x = \frac{1 - \cos 2x}{2}$.
- $\cos^2 x = \frac{1 + \cos 2x}{2}$.
- $\operatorname{tg}^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$.
- $\operatorname{ctg}^2 x = \frac{1 + \cos 2x}{1 - \cos 2x}$.
- $\sin x \pm \sin y = 2 \sin \frac{x \pm y}{2} \cos \frac{x \mp y}{2}$.
- $\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$.
- $\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$.
- $\operatorname{tg} x \pm \operatorname{tg} y = \frac{\sin(x \pm y)}{\cos x \cos y}$.
- $\operatorname{ctg} x \pm \operatorname{ctg} y = \frac{\sin(y \pm x)}{\sin x \sin y}$.
- $A \sin x + B \cos x = \sqrt{A^2 + B^2} \sin(x + \varphi)$, где $\sin \varphi = \frac{B}{\sqrt{A^2+B^2}}$, а $\cos \varphi = \frac{A}{\sqrt{A^2+B^2}}$.
- $\arcsin x + \arccos x = \frac{\pi}{2}$.
- $\operatorname{arctg} x + \operatorname{arcctg} x = \frac{\pi}{2}$.
- $\arcsin x = \operatorname{arctg} \frac{x}{\sqrt{1-x^2}}$.
- $\arccos x = 2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}}$.
- $\operatorname{arctg} x = \arcsin \frac{x}{\sqrt{1+x^2}}$.
- $\operatorname{arcctg} x = \begin{cases} \arcsin \frac{1}{\sqrt{1+x^2}}, & x \geqslant 0, \\ \pi - \arcsin \frac{1}{\sqrt{1+x^2}}, & x < 0. \end{cases}$

1.2 Hyperbolic Identities

- $\operatorname{sh} x = \frac{e^x - e^{-x}}{2}$.
- $\operatorname{ch} x = \frac{e^x + e^{-x}}{2}$.
- $\operatorname{ch}^2 x - \operatorname{sh}^2 x = 1$.
- $\operatorname{sh}(x \pm y) = \operatorname{sh} x \operatorname{ch} y \pm \operatorname{sh} y \operatorname{ch} x$.
- $\operatorname{ch}(x \pm y) = \operatorname{ch} x \operatorname{ch} y \pm \operatorname{sh} x \operatorname{sh} y$.
- $\operatorname{th}(x \pm y) = \frac{\operatorname{th} x \pm \operatorname{th} y}{1 \pm \operatorname{th} x \operatorname{th} y}$.
- $\operatorname{cth}(x \pm y) = \frac{1 \pm \operatorname{cth} x \operatorname{cth} y}{\operatorname{cth} x \pm \operatorname{cth} y}$.
- $\operatorname{sh} 2x = 2 \operatorname{sh} x \operatorname{ch} x$.
- $\operatorname{ch} 2x = \operatorname{ch}^2 x + \operatorname{sh}^2 x$.
- $\operatorname{th} 2x = \frac{2 \operatorname{th} x}{1 + \operatorname{th}^2 x}$.
- $\operatorname{cth} 2x = \frac{1}{2}(\operatorname{th} x + \operatorname{cth} x)$.
- $\operatorname{sh} x \operatorname{sh} y = \frac{\operatorname{ch}(x+y) - \operatorname{ch}(x-y)}{2}$.
- $\operatorname{sh} x \operatorname{ch} y = \frac{\operatorname{sh}(x+y) + \operatorname{sh}(x-y)}{2}$.
- $\operatorname{ch} x \operatorname{ch} y = \frac{\operatorname{ch}(x+y) + \operatorname{ch}(x-y)}{2}$.
- $\operatorname{sh} x \pm \operatorname{sh} y = 2 \operatorname{sh} \frac{x \pm y}{2} \operatorname{ch} \frac{x \mp y}{2}$.
- $\operatorname{ch} x + \operatorname{ch} y = 2 \operatorname{ch} \frac{x+y}{2} \operatorname{ch} \frac{x-y}{2}$.
- $\operatorname{ch} x - \operatorname{ch} y = 2 \operatorname{sh} \frac{x+y}{2} \operatorname{sh} \frac{x-y}{2}$.
- $\operatorname{sh}^2 x = \frac{\operatorname{ch} 2x - 1}{2}$, $\operatorname{ch}^2 x = \frac{\operatorname{ch} 2x + 1}{2}$.